

Misalignment of the AC Final Orbits

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November 16, 2012

1 Background

This report is primarily motivated by the biases and scatter in the AC Final orbit X- and Y-rotations (RX and RY), as illustrated for ESA, NGS and SIO in Figure 1. Additional motivation is provided by rather surprising results presented at IGS2010, EGU and IGS2012 by Jim Ray and his colleagues (2010, 2011, and 2012). They showed that the Final orbits (IGS) are rotationally misaligned from the Rapid orbits (IGR) and from the observed part (IGA) of the Ultra-rapid orbits (IGU), especially in RX and RY. Unexpectedly, the IGR and IGA are more consistent with one another and more accurate than the IGS. A few ideas were proposed to explain the results, including the possibility that procedural issues exist in the Final orbit combination.

At the AC level, the Ultra-rapid and Rapid procedures transfer the terrestrial frame to the orbits by tightly fixing the *a priori* positions of the IGB08 reference frame stations. The Final procedures use a no-net rotation (or other removable) constraint, satisfied over the IGB08 core network. It is incumbent upon the ACs to ensure that their procedures are consistent with the expectations of the combinations. At the combination level, the main procedural difference between the IGS and IGR/IGU is that AC SINEX and AC ERP X- and Y-rotations are applied for the IGS. This report summarizes a review of the Final orbit combination model. In short, two issues were found: the signs used for the AC SINEX X- and Y-rotations are incorrect and the AC ERP rotations are no longer necessary.

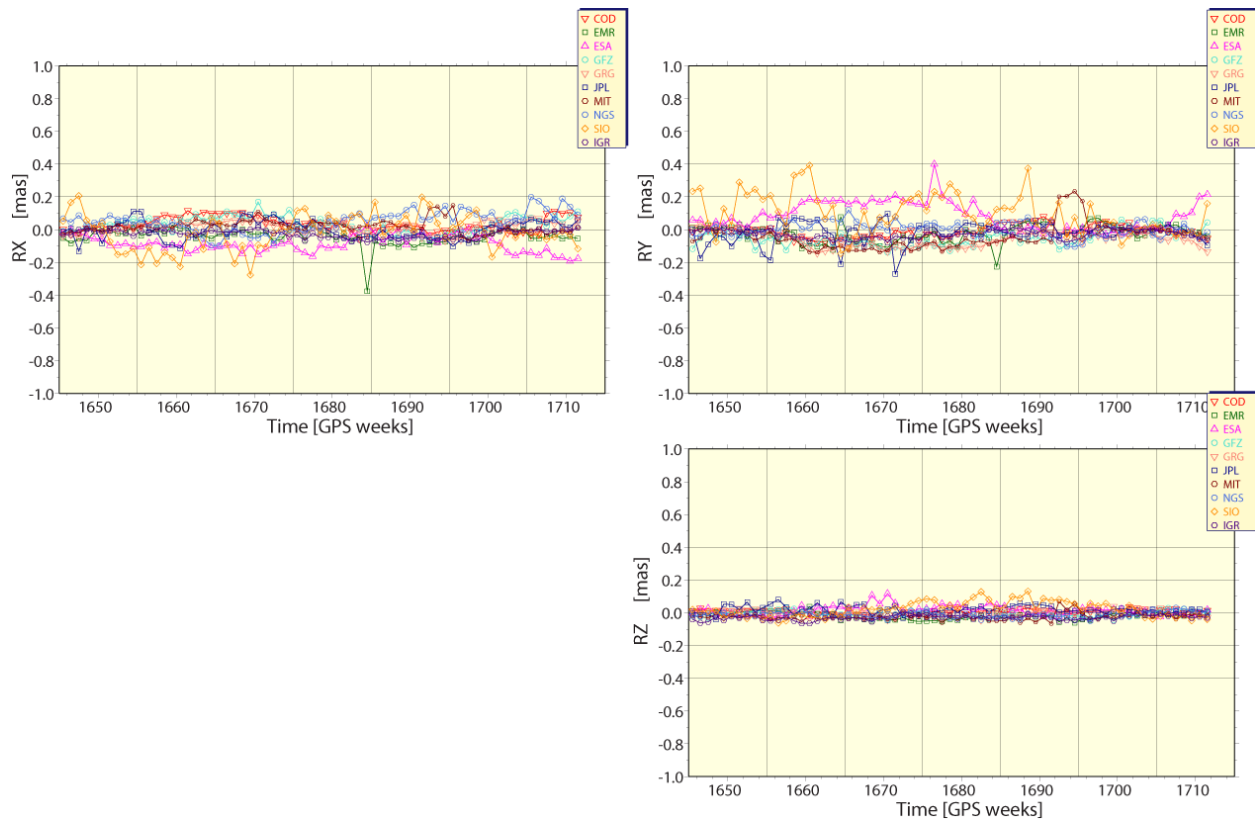


Figure 1. Rotations of AC orbital frames w.r.t. the IGS Final combined orbit, averaged weekly.

2 Final Orbit Combination Model

The rotations in Figure 1 were estimated during the routine operational IGS Final orbit combinations. The model used for those combinations is consistent with the following equations:

$$X_i^{\text{igs}} = X_i^{\text{ac}} + \text{TX}^{\text{ac}} + D^{\text{ac}} \cdot X_i^{\text{ac}} + \tilde{\text{RZ}}^{\text{ac}} \cdot Y_i^{\text{ac}} - \tilde{\text{RY}}^{\text{ac}} \cdot Z_i^{\text{ac}} \quad (1a)$$

$$Y_i^{\text{igs}} = Y_i^{\text{ac}} + \text{TY}^{\text{ac}} + D^{\text{ac}} \cdot Y_i^{\text{ac}} - \tilde{\text{RZ}}^{\text{ac}} \cdot X_i^{\text{ac}} + \tilde{\text{RX}}^{\text{ac}} \cdot Z_i^{\text{ac}} \quad (1b)$$

$$Z_i^{\text{igs}} = Z_i^{\text{ac}} + \text{TZ}^{\text{ac}} + D^{\text{ac}} \cdot Z_i^{\text{ac}} + \tilde{\text{RY}}^{\text{ac}} \cdot X_i^{\text{ac}} - \tilde{\text{RX}}^{\text{ac}} \cdot Y_i^{\text{ac}} \quad (1c)$$

where, X_i^{igs} , Y_i^{igs} and Z_i^{igs} are the combined geocentric positions of satellite i ; X_i^{ac} , Y_i^{ac} and Z_i^{ac} are the AC geocentric positions of satellite i ; TX^{ac} , TY^{ac} and TZ^{ac} are the estimated origin translations of the AC orbital frame; D^{ac} is the estimated change in scale of the AC orbital frame; $\tilde{\text{RX}}^{\text{ac}}$, $\tilde{\text{RY}}^{\text{ac}}$ and $\tilde{\text{RZ}}^{\text{ac}}$ are *counter-clockwise* positive rotations of the AC orbital frame about the X-, Y- and Z-axes, respectively. The rotations $\tilde{\text{RX}}^{\text{ac}}$, $\tilde{\text{RY}}^{\text{ac}}$ and $\tilde{\text{RZ}}^{\text{ac}}$ contain the following parts:

$$\tilde{\text{RX}}^{\text{ac}} = \text{RX}^{\text{ac}} + \text{RX}_{\text{snx}}^{\text{ac}} + \text{RX}_{\text{erp}}^{\text{ac}} \quad (2a)$$

$$\tilde{\text{RY}}^{\text{ac}} = \text{RY}^{\text{ac}} + \text{RY}_{\text{snx}}^{\text{ac}} + \text{RY}_{\text{erp}}^{\text{ac}} \quad (2b)$$

$$\tilde{\text{RZ}}^{\text{ac}} = \text{RZ}^{\text{ac}} - \text{RZ}_{\text{snx}}^{\text{ac}} \quad (2c)$$

RX^{ac} , RY^{ac} and RZ^{ac} are estimated in the orbit combination (e.g. Figure 1). $\text{RX}_{\text{snx}}^{\text{ac}}$, $\text{RY}_{\text{snx}}^{\text{ac}}$ and $\text{RZ}_{\text{snx}}^{\text{ac}}$ are from the IGS SINEX combination. According to P. Rebischung, the model for that combination is consistent with the following equations:

$$x_j^{\text{i08}} = x_j^{\text{ac}} + \text{TX}_{\text{snx}}^{\text{ac}} + D_{\text{snx}}^{\text{ac}} \cdot x_j^{\text{ac}} - \text{RZ}_{\text{snx}}^{\text{ac}} \cdot y_j^{\text{ac}} + \text{RY}_{\text{snx}}^{\text{ac}} \cdot z_j^{\text{ac}} \quad (3a)$$

$$y_j^{\text{i08}} = y_j^{\text{ac}} + \text{TY}_{\text{snx}}^{\text{ac}} + D_{\text{snx}}^{\text{ac}} \cdot y_j^{\text{ac}} + \text{RZ}_{\text{snx}}^{\text{ac}} \cdot x_j^{\text{ac}} - \text{RX}_{\text{snx}}^{\text{ac}} \cdot z_j^{\text{ac}} \quad (3b)$$

$$z_j^{\text{i08}} = z_j^{\text{ac}} + \text{TZ}_{\text{snx}}^{\text{ac}} + D_{\text{snx}}^{\text{ac}} \cdot z_j^{\text{ac}} - \text{RY}_{\text{snx}}^{\text{ac}} \cdot x_j^{\text{ac}} + \text{RX}_{\text{snx}}^{\text{ac}} \cdot y_j^{\text{ac}} \quad (3c)$$

where, x_j^{i08} , y_j^{i08} and z_j^{i08} are the IGB08 geocentric positions of each core reference frame station j ; x_j^{ac} , y_j^{ac} and z_j^{ac} are the geocentric positions of each core station j taken from the AC SINEX files; $\text{TX}_{\text{snx}}^{\text{ac}}$, $\text{TY}_{\text{snx}}^{\text{ac}}$ and $\text{TZ}_{\text{snx}}^{\text{ac}}$ are the estimated origin translations of the AC terrestrial frame; $D_{\text{snx}}^{\text{ac}}$ is the estimated change in the scale of the AC terrestrial frame; and $\text{RX}_{\text{snx}}^{\text{ac}}$, $\text{RY}_{\text{snx}}^{\text{ac}}$ and $\text{RZ}_{\text{snx}}^{\text{ac}}$ are estimated *clockwise* positive rotations of the AC terrestrial frame. The convention used for these rotations presents an issue for $\text{RX}_{\text{snx}}^{\text{ac}}$ and $\text{RY}_{\text{snx}}^{\text{ac}}$ that is discussed in more detail shortly. Let us first define and discuss the final two variables in the combination model.

$\text{RX}_{\text{erp}}^{\text{ac}}$ and $\text{RY}_{\text{erp}}^{\text{ac}}$ are rotations derived from x and y polar motion (PM_x, PM_y) differences between the AC and combined IGS ERPs from the SINEX combination after removing the AC SINEX rotation, i.e.:

$$RX_{erp}^{ac} = PMy^{ac} + RX_{snx}^{ac} - PMy^{igs} \quad (4a)$$

$$RY_{erp}^{ac} = PMx^{ac} + RY_{snx}^{ac} - PMx^{igs} \quad (4b)$$

The AC SINEX and ERP rotations were added to the combination model by T. Springer in early 2000 (see IGS Mail #2750; igsb.jpl.nasa.gov/pipermail/igsmail/2000/002824.html). The introduction of RX_{snx}^{ac} , RY_{snx}^{ac} and RZ_{snx}^{ac} was proposed by Kouba et al. (1998) to maintain consistency between the IGS SINEX and Final orbit combinations. Before that, there was no combination of AC SINEX files. The approach is predicated upon the assumption that each set of AC Final orbits, ERPs, SINEX, and clocks are each internally self-consistent. Provided the assumption is valid, then applying RX_{snx}^{ac} , RY_{snx}^{ac} and RZ_{snx}^{ac} ensures long-term consistency between the IGS terrestrial frame and the IGS Final orbits. RX_{erp}^{ac} and RY_{erp}^{ac} were introduced by T. Springer to approximate day-to-day variations in the AC orbits not captured by the AC weekly SINEX rotations. At that time, the IGS SINEX combinations were based on weekly integrations, and so the associated RX_{snx}^{ac} , RY_{snx}^{ac} and RZ_{snx}^{ac} were weekly averages. In his report on these matters, T. Springer showed that the ERP rotations significantly reduced the AC rotational biases and scatter. Since Wk 1702, however, the SINEX combinations are based on daily integrations, and so there is no longer a need to include the ERP rotations in the combination model.

Returning now to the issue mentioned above. The ACC procedures currently use the clockwise positive AC SINEX rotations from Eqs. (3) to align the AC orbits using Eqs. (1) *without* reversing the sign for RX_{snx}^{ac} and RY_{snx}^{ac} . However, the sign used for RZ_{snx}^{ac} is currently the correct one, as shown in Eq. (2c), and does not need to be changed. The erroneous signs on RX_{snx}^{ac} and RY_{snx}^{ac} must be reversed. Thus, the following corrected model:

$$X_i^{igs} = X_i^{ac} + TX^{ac} + D^{ac} \cdot X_i^{ac} + (RZ^{ac} - RZ_{snx}^{ac}) \cdot Y_i^{ac} - (RY^{ac} - RY_{snx}^{ac}) \cdot Z_i^{ac} \quad (5a)$$

$$Y_i^{igs} = Y_i^{ac} + TY^{ac} + D^{ac} \cdot Y_i^{ac} - (RZ^{ac} - RZ_{snx}^{ac}) \cdot X_i^{ac} + (RX^{ac} - RX_{snx}^{ac}) \cdot Z_i^{ac} \quad (5b)$$

$$Z_i^{igs} = Z_i^{ac} + TZ^{ac} + D^{ac} \cdot Z_i^{ac} + (RY^{ac} - RY_{snx}^{ac}) \cdot X_i^{ac} - (RX^{ac} - RX_{snx}^{ac}) \cdot Y_i^{ac} \quad (5c)$$

should definitely be adopted for the Final orbits prior to the start of repro2 combinations, which could begin sometime in early 2013. But the new model really is needed sooner so that the Final orbits are not unnecessarily harmed by the ERP rotations. Therefore, it is recommended to adopt Eqs. (5) starting with Wk 1716 (to be combined Dec. 14 or 15, 2012). At that time, the 3-day solution from the CODE group (currently called COD) should be dropped (or replaced by COF, using the COD moniker) because the combinations would no longer properly handle non-daily Final products.

3 Results from the Modified Combination Model

For your entertainment, test combinations for GPS Wks 1702 through 1710 were performed using the corrected Final orbit combination in Eqs. (5), and then compared to operational results (Table 1).

As should be expected, most of the changes occurred in RX and RY. The scale and translations, TX, TY and TZ are effectively unchanged. The WRMS agreement decreases by small amounts ($\sim 0.8 \pm 0.5$ mm) for nearly all ACs; JPL has a ~ 1 mm increase. There are large decreases in the RX and RY scatters for nearly all ACs. The new AC rotation biases probably reflect effects of the AC orbit modeling differences and/or possibly some other modeling choices. These should get more attention in the future, but they probably vary with time so we need to wait until repro2 is done. The only

standout bias is for ESA (RX = 97 μ as), with smaller offsets for GFZ (RX = -70 μ as) and NGS (RX and RY, 40 to 55 μ as).

With the new method, the RX, RY and RZ scatters are nearly all between 25 and 35 μ as (i.e., \sim 4 mm rotational variation). The exceptions are for ESA (RY rms \sim 55 μ as) and GFZ (RX and RY rms both \sim 60 μ as) on the high side, and MIT on the low side (RX and RY rms are 19 to 20 μ as). The reasons for these differences should be understood. For instance, does the MIT approach for constraining once-per-revolution solar radiation terms help (artificially??) stabilize their solutions?

With respect to IGR, the magnitude of the RX and RY biases increased; the scatter in RX improved, and the scatter in RY got slightly worse. To know whether this change resolves the misalignment between IGS and IGR reported by Ray et al. (2010, 2011 & 2012) requires further study. However, recall that the IGR orbits inherit the terrestrial frame from the weighted average of the AC orbits, and thus depends on whether ACs with the heaviest weight (currently ESA) affix the IGB08 reference frame stations to their *a priori* values. It is unclear from descriptions of current ESA practices whether this happens. Also, provided the AC Final products are indeed each internally self-consistent, then the application of the AC SINEX rotations at the combination level should align the combined IGS very closely to IGB08. Given the conditions that must be met for very good IGS \rightarrow IGR agreement, it is easy to imagine different ways to generate a rotational misalignment. Therefore, a rotational misalignment may still exist despite the corrections to the Final orbit combination model. And, thus, this issue should be study further using a longer span of new results.

In summary, the results from test combinations indicate that the corrected combination model makes important improvements in nearly all AC RX and RY rotations. The Final orbits may now be more closely aligned to IGR, but that should be studied later. These results all support the recommendation from section 2, that is, Eqs. (5) should be adopted as soon as possible, preferably after the Fall AGU in San Francisco. This would mean that products starting with Wk 1716 would use the new model.

Table 1. Combination results averaged over GPS Wks 1702 through 1710. In (A), the operational combination was used. In (B), the modified model from Eqs. (5) was used. IGR was included in the combinations for comparison only. COD and SIO were omitted—they provided weekly SINEX files and so there are no daily SINEX rotations to apply to their orbits. COF was available only since Wk 1706.

	TX [mm]	TY [mm]	TZ [mm]	RX [uas]	RY [uas]	RZ [uas]	SCL [ppb]	RMS [mm]	WRMS [mm]
cof old	-0.17	0.17	-0.43	54.80	-78.57	18.60	-0.456	12.09	11.89
+ / -	0.75	0.75	1.44	55.86	58.89	16.66	0.044	1.60	1.53
cof new	-0.06	0.17	-0.69	19.51	11.00	18.03	-0.431	10.49	10.17
+ / -	0.73	0.71	1.57	30.33	33.36	15.79	0.036	1.12	1.04
emr old	0.23	0.00	2.89	-40.86	-0.69	-10.74	0.278	17.86	17.77
+ / -	1.78	1.46	2.01	32.35	36.97	16.26	0.043	0.91	0.84
emr new	0.34	-0.09	2.77	-22.23	8.77	-8.09	0.305	16.66	16.60
+ / -	1.75	1.54	1.94	27.29	23.81	17.04	0.036	1.39	1.35
esa old	0.17	-0.09	0.49	-159.91	104.23	-8.20	0.040	7.80	7.74
+ / -	0.38	0.37	1.04	68.61	86.23	15.85	0.042	1.05	1.15
esa new	0.23	-0.06	0.46	97.26	-31.80	-9.40	0.059	7.17	7.09
+ / -	0.43	0.24	0.95	34.13	55.17	15.65	0.034	0.82	0.74
gfz old	-0.83	0.29	-2.00	61.26	-20.57	17.26	-0.433	12.34	11.91
+ / -	0.92	0.71	1.96	72.70	63.28	17.14	0.039	1.98	2.11
gfz new	-0.77	0.23	-1.71	-70.09	-35.49	16.60	-0.413	12.06	11.86
+ / -	0.91	0.69	2.04	63.16	58.94	16.93	0.031	1.86	1.90
grg old	1.26	0.69	5.00	41.20	-68.43	-3.49	0.415	15.71	15.00
+ / -	1.17	0.83	2.28	44.90	46.03	23.65	0.052	1.18	1.08
grg new	1.14	0.83	5.57	-11.69	15.63	-3.34	0.428	15.40	14.80
+ / -	1.22	0.86	2.21	26.23	35.63	24.64	0.042	1.09	0.99
igr old	0.03	-0.11	-1.34	-4.40	-35.00	-19.20	-0.180	4.94	4.77
+ / -	0.30	0.32	0.91	28.86	29.10	14.61	0.029	0.54	0.60
igr new	0.09	-0.09	-1.23	-16.09	41.31	-19.66	-0.161	4.43	4.17
+ / -	0.28	0.28	0.84	26.75	32.64	14.95	0.023	0.50	0.38
jpl old	0.00	-0.86	-2.17	43.37	-3.51	8.46	0.405	15.60	15.34
+ / -	1.28	1.22	1.93	55.54	50.53	22.63	0.041	0.95	1.03
jpl new	-0.03	-1.06	-1.83	-6.54	15.71	8.77	0.423	14.77	14.51
+ / -	1.18	1.24	1.96	27.89	26.65	23.21	0.034	1.00	1.12
mit old	0.00	-0.20	-2.74	-15.03	-18.23	-17.71	-0.206	7.71	7.37
+ / -	0.00	0.41	1.01	34.97	32.54	22.09	0.032	1.56	1.44
mit new	0.00	-0.17	-2.54	-29.46	0.97	-17.34	-0.187	6.86	6.51
+ / -	0.00	0.45	0.98	18.99	21.92	21.61	0.027	1.14	1.04
ngs old	-0.37	-0.06	2.94	141.49	-3.20	15.63	0.295	11.34	9.54
+ / -	0.94	0.87	2.75	74.54	48.57	24.67	0.065	0.94	0.85
ngs new	-0.31	0.09	3.03	-55.43	41.43	15.29	0.317	10.94	9.00
+ / -	0.93	0.89	2.68	40.52	38.87	25.32	0.054	0.84	0.91

Acknowledgements

Thanks to Jim Ray for reviewing this report, which improved significantly from his comments. Personal communications with Tim Springer, Rémi Ferland, Yves Mireault, and Paul Rebischung, along with open discussions amongst the ACs over the IGS-ACS email exploder also contributed to improving the contents of this report.

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