A Kalman Filter to Combine VLBI UT1 & GPS LOD Estimates

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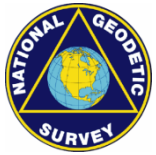

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OUTLINE

- Context & objectives
- Difficulties with satellite-based LOD
- Kalman combination filter model & results
- Compare new KF series with other combinations
- Correlations with atmosphere angular momentum (AAM) excitation
- Fortnightly, monthly, & 9-d bands
- Check consistency of UT1 & LOD values & power spectra
- Conclusions

Context & Objectives

- Cannot easily assimilate full UT1 & LOD information in ITRF time series combination with station coordinates & other EOPs
  - UT1/LOD from 24-hr multi-baseline VLBI included OK
  - UT1 from 1-hr single-baseline VLBI ***not*** included
    - not available in SINEX format
    - can distort station positions due to limited observing geometry
  - LOD from satellite techniques also ***not*** included
    - time-varying biases are significant & not easily modeled
    - frame-related errors are not significant compared to orbit-related biases
- Previously proposed a multi-step ITRF & EOP combination process
  - ITRF2005-type TRF + EOP combination
  - Reduce 1-hr single-baseline VLBI sessions consistent with ITRF2005 for denser UT1 time series
  - Merge UT1 time series from ITRF & 1-hr VLBI steps
  - Assimilate GPS LOD into VLBI UT1 time series using Kalman filter
- Question: Can a Kalman filter provide a useful combination of VLBI UT1 and satellite LOD quasi-optimally?

Difficulties with Direct LOD Combination

- Satellite-based LOD estimates are biased *(Ray, 1996)*
  - biases are quasi-stochastic but correlated in time
  - also correlated among IGS Analysis Centers
  - biases mostly reflect errors in orbit modeling (i.e., constellation-averaged drift of ascending nodes)
  - GPS LODs also show artifactual alias signals (e.g., in fortnightly band)
- Other combinations sometimes ignore GPS biases
  - *Thaller et al. (2007)* assumed constant LOD biases & used only 2 weeks of continuous VLBI & GPS data
  - ignored all other biases too
  - their approach can not be applied when VLBI data has gaps
  - applied smoothing filter using continuous linear segment parameterization (also distorts signal content)
  - did not compare with any independent observations; C04 comparison is not independent
- JPL group has long used Kalman filter for EOP combinations
  - *Morabito et al. (1988)* & *Gross et al. (1998)*
  - copes well with natural stochastic excitation
  - but not used to model satellite-based LOD biases

Kalman Filter Combination Model

- UT1 is the (negative) integral of LOD + random walk
  - excitation variance using modern data found to agree with *Morabito et al. (1988)* value

$$-\frac{d^2}{dt^2}UT1R = \frac{1}{86400}\frac{d}{dt}LODR = w_L \qquad \sigma^2_{w_L} = 3.6\ \mu\mathrm{s}^2/\mathrm{day}^3$$

- Gauss-Markov process used to model GPS LOD biases
  - time-constant $= 1/\beta = 2.17$ days

$$\frac{d}{dt}B_M = -\beta B_M + w_M \qquad \sigma^2_{w_M} = 56.4\beta\ \mu\mathrm{s}^2/\mathrm{day}^3$$

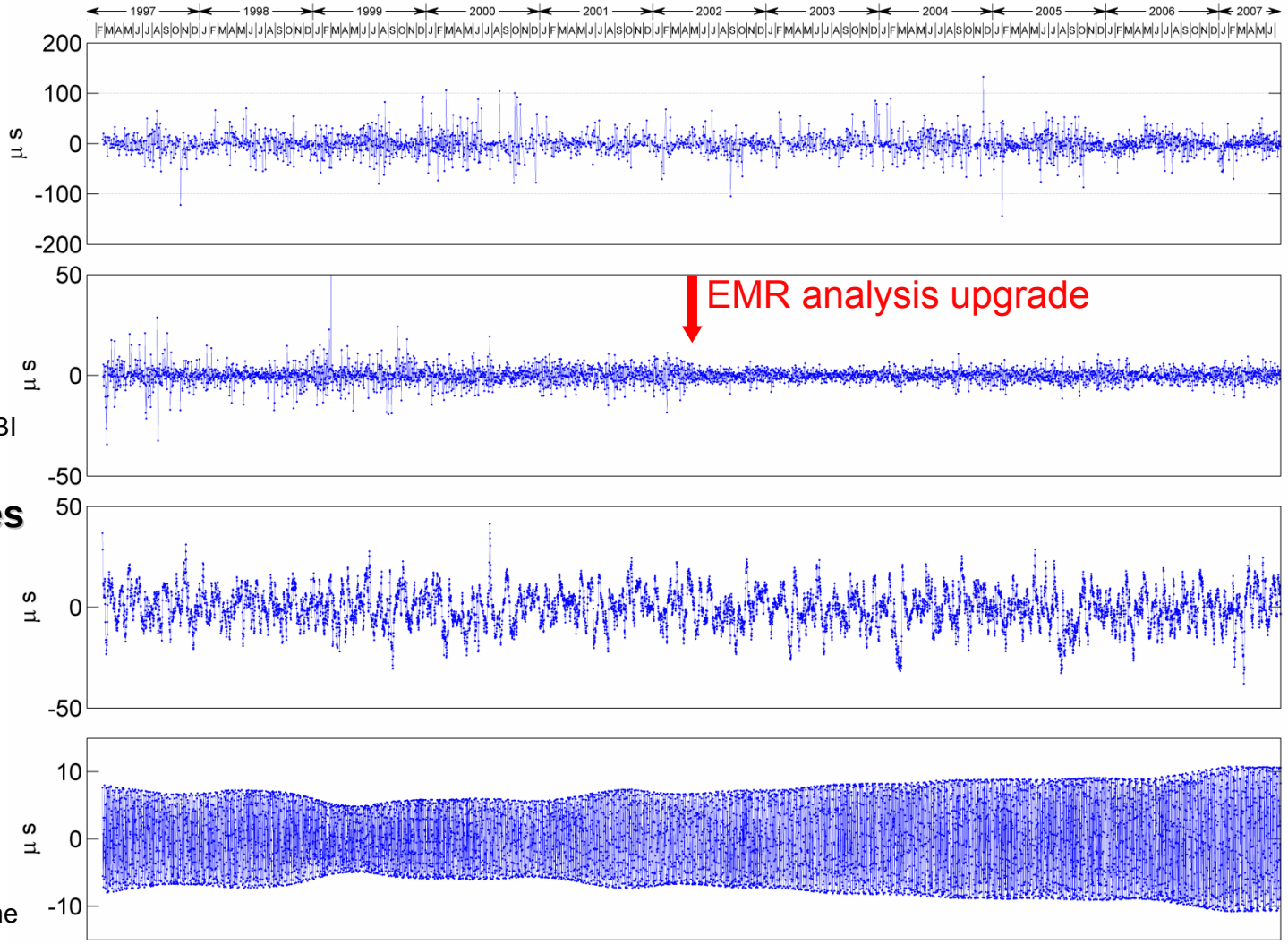
- Harmonic with period 14.19 d added to capture effect of mismodeled tides in GPS LOD biases *(Kouba, 2003)*

Input Data Sets

- UT1 from 24-hr multi-baseline VLBI sessions
  - series “2007c” from NASA/GSFC
  - from 21 Feb 1997 to 17 Jul 2007
  - at irregular epochs, about 2 to 3 per week
  - formal errors scaled by 2
- UT1 from 1-hr single-baseline VLBI sessions
  - series “int21” from NASA/GSFC
  - from 21 Feb 1997 to 18 Jul 2007
  - formal errors scaled by 2
  - at irregular epochs, about 5 per week
  - consistent with 24-hr sessions: mean differences = -0.7 ± 22.5 μs with N = 1244 & χ^2/dof = 2.58
- daily LOD from IGS combination
  - series “igs00p03.erp”
  - noon epochs from 23 Feb 1997 to 18 Jul 2007
  - formal errors scaled by 2
  - some bias corrections applied already by IGS using IERS Bulletin A
- corrections for zonal tides applied to all series before combination
- VLBI UT1 accuracy could be improved by adding GPS polar motion & global network in raw reduction *(Ray et al., 2005)* – not studied here

Some Characteristics of Kalman Filter Output

- **VLBI (1-hr) UT1 residuals**
  - white over full frequency range
- **GPS LOD residuals**
  - approximately white
  - with small peak at 13.7 d
  - possible difference in *a priori* tidal models wrt VLBI
- **Gauss-Markov values for GPS LOD biases**
  - peak-to-peak range = ± 40 μs
  - RMS = 9 μs
- **14.19-d periodic**
  - treated as GPS artifact
  - amplitude varies between 5 & 11 μs
  - phase varies linearly w/ time due to changing period



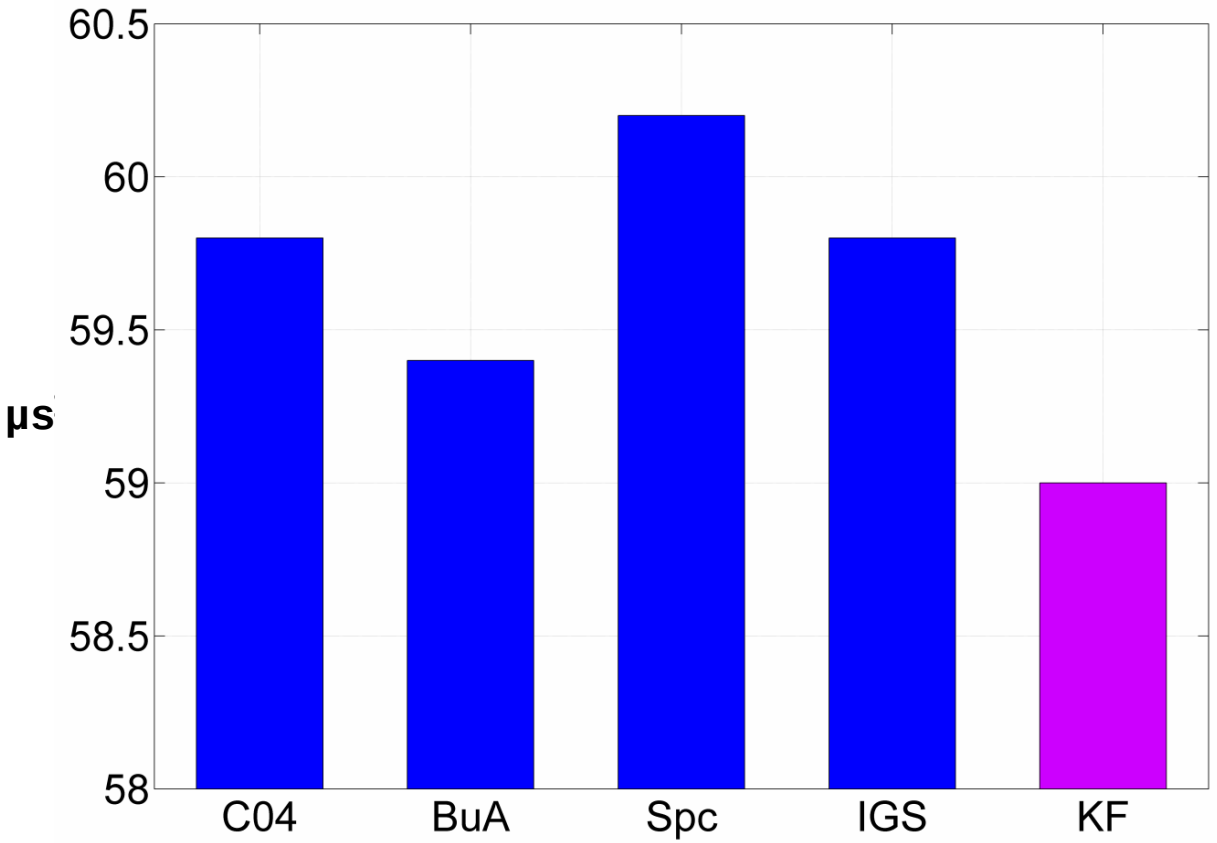
Compare LOD w/ AAM Excitation

- our KF w/ other LOD combinations
  - corrected for zonal tides, LODS *(Yoder et al., 1981; Kantha et al., 1998)*
- Atmospheric Angular Momentum (AAM) from NCEP Reanalysis
  - 4 values daily, during Feb 1997 – Dec 2006
  - inverted barometer correction applied
  - averaged to daily values at 00:00 or 12:00 epochs to match respective LOD series epochs
- for each (LODS – AAM) time series, fit for imperfectly known geophysical & systematic effects *(Kouba & Vondrak, 2005)*
  - annual + semi-annual differences
  - monthly (27.56 d) oceanic tide correction
  - fortnightly (13.63/13.66 d) oceanic tide corrections
  - k/C core-mantle coupling constant
  - long-term drift differences
  - AAM transfer function scale factor
- compute residuals & compare
- compute LODS/AAM cross-correlations & compare

(LODS – AAM) RMS Residuals

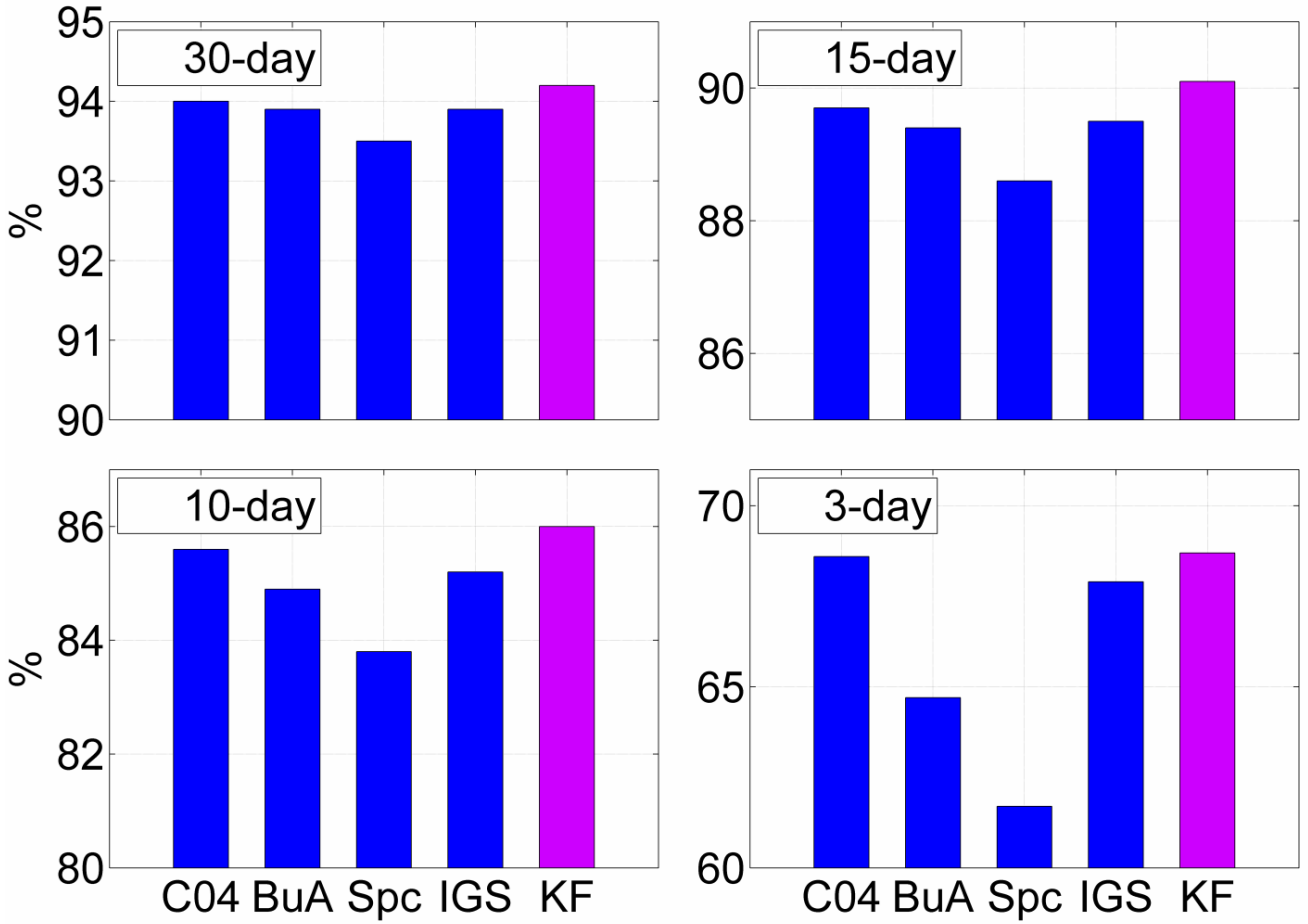
- LOD time series studied
  - IERS 05C04 (00:00)
  - IERS Bulletin A (00:00)
  - JPL's SPACE 2006 (12:00)
  - IGS (no UT1) (12:00)
  - our KF (12:00)

- our KF has smallest residual



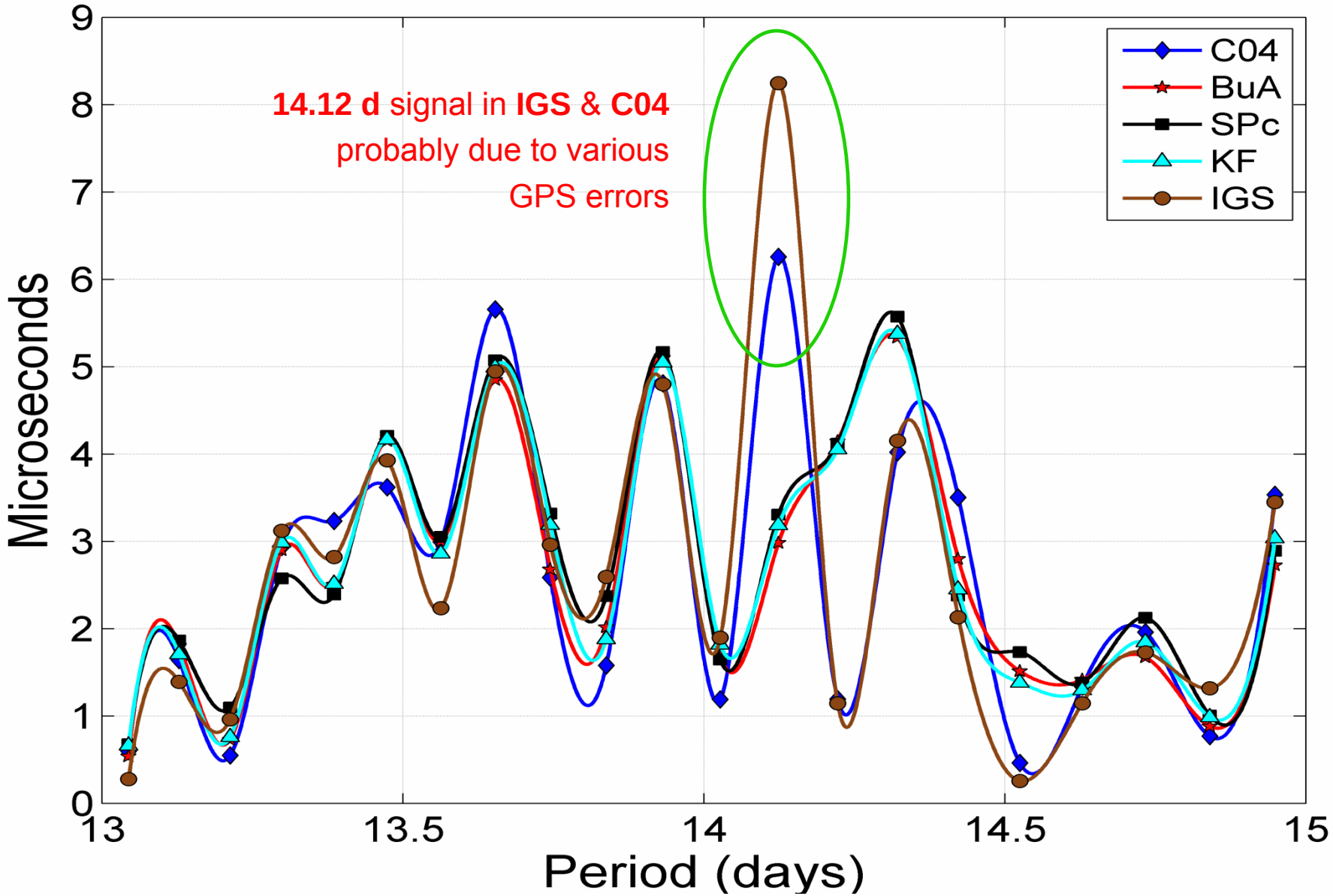
LODS/AAM Correlation Coefficients

- computed over sliding windows from 3 d to 5.6 yr
  - correlation over full range = 98.9% for all series

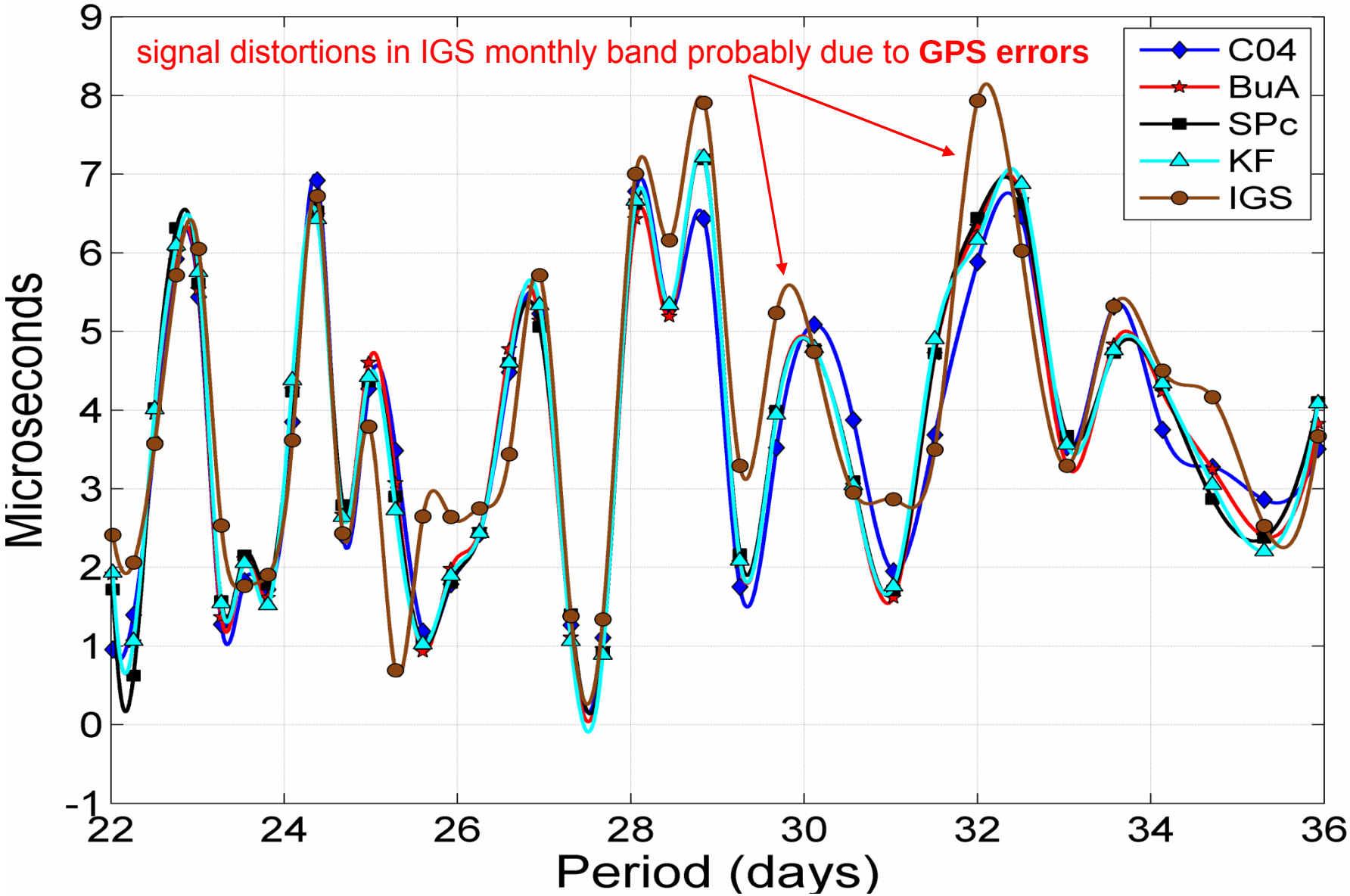


- KF has highest correlations w/ AAM over all intervals; SPACE 2006 has lowest

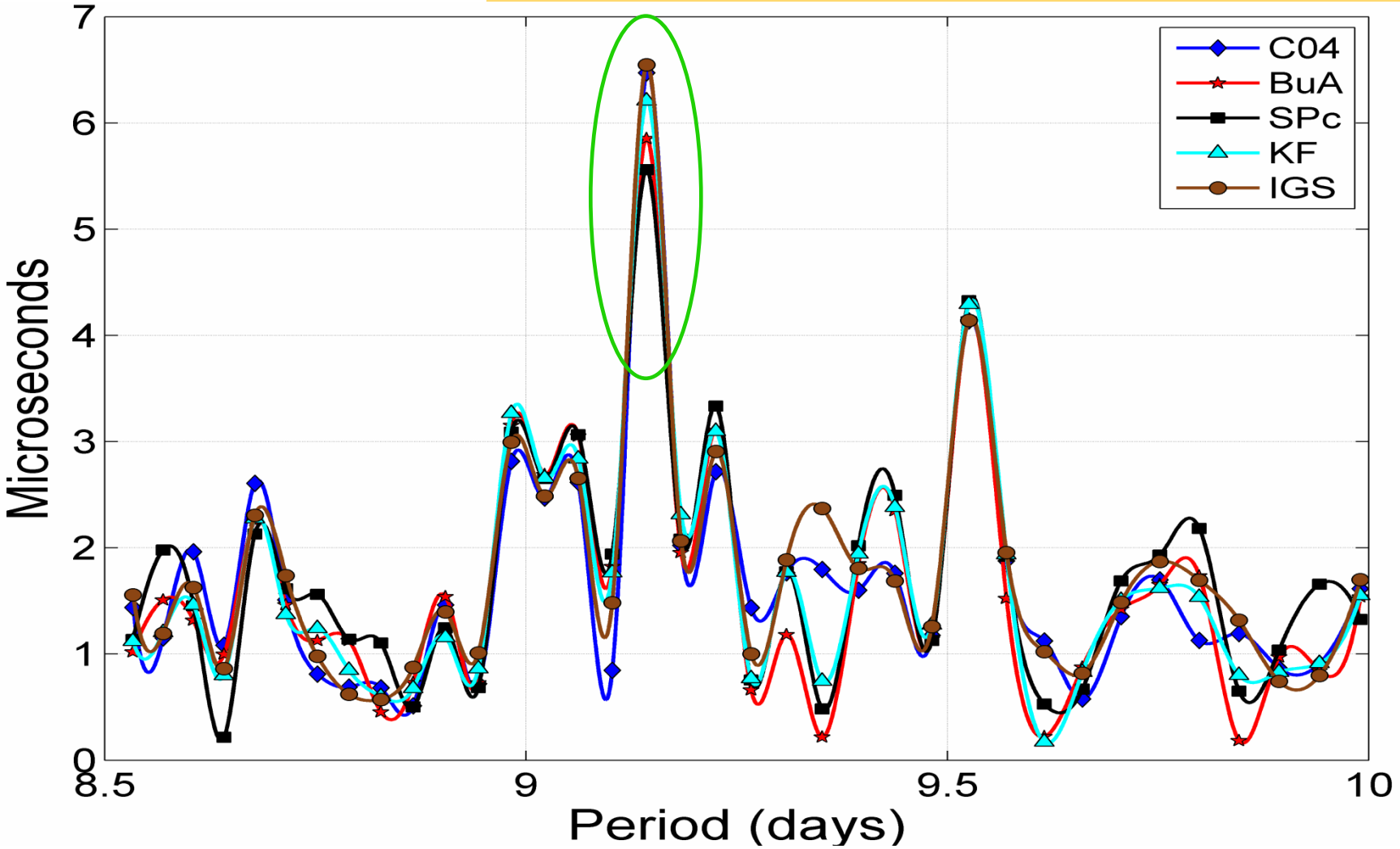
Fortnightly Band – Spurious IGS LOD Peak



Monthly Band – Probable GPS Errors



9-d Band – Unmodeled Geophysical Effect?

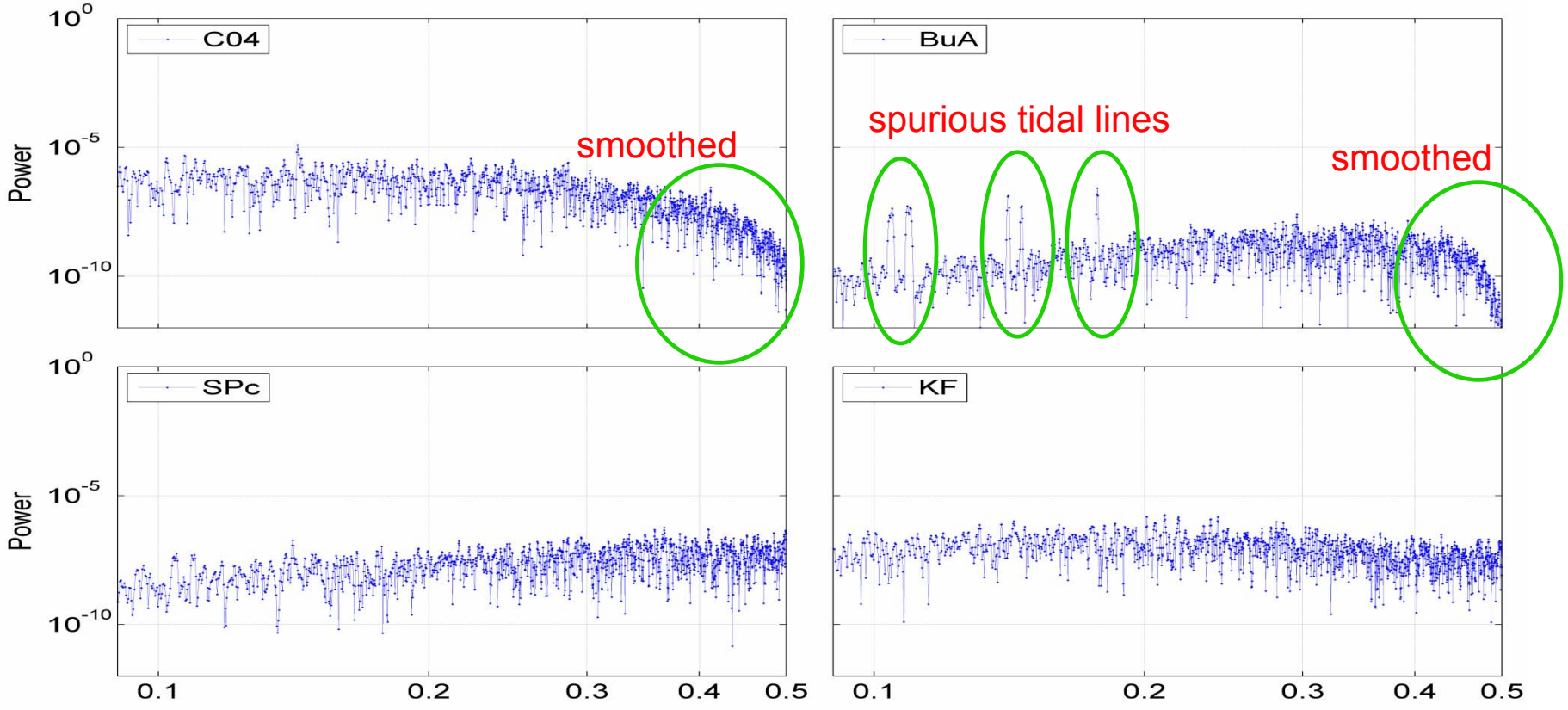


- 9.14 d peak seen in all LODS series – probably geophysical
- needs further investigation – could be included in (LODS-AAM) fit

Consistency of UT1 & LOD values

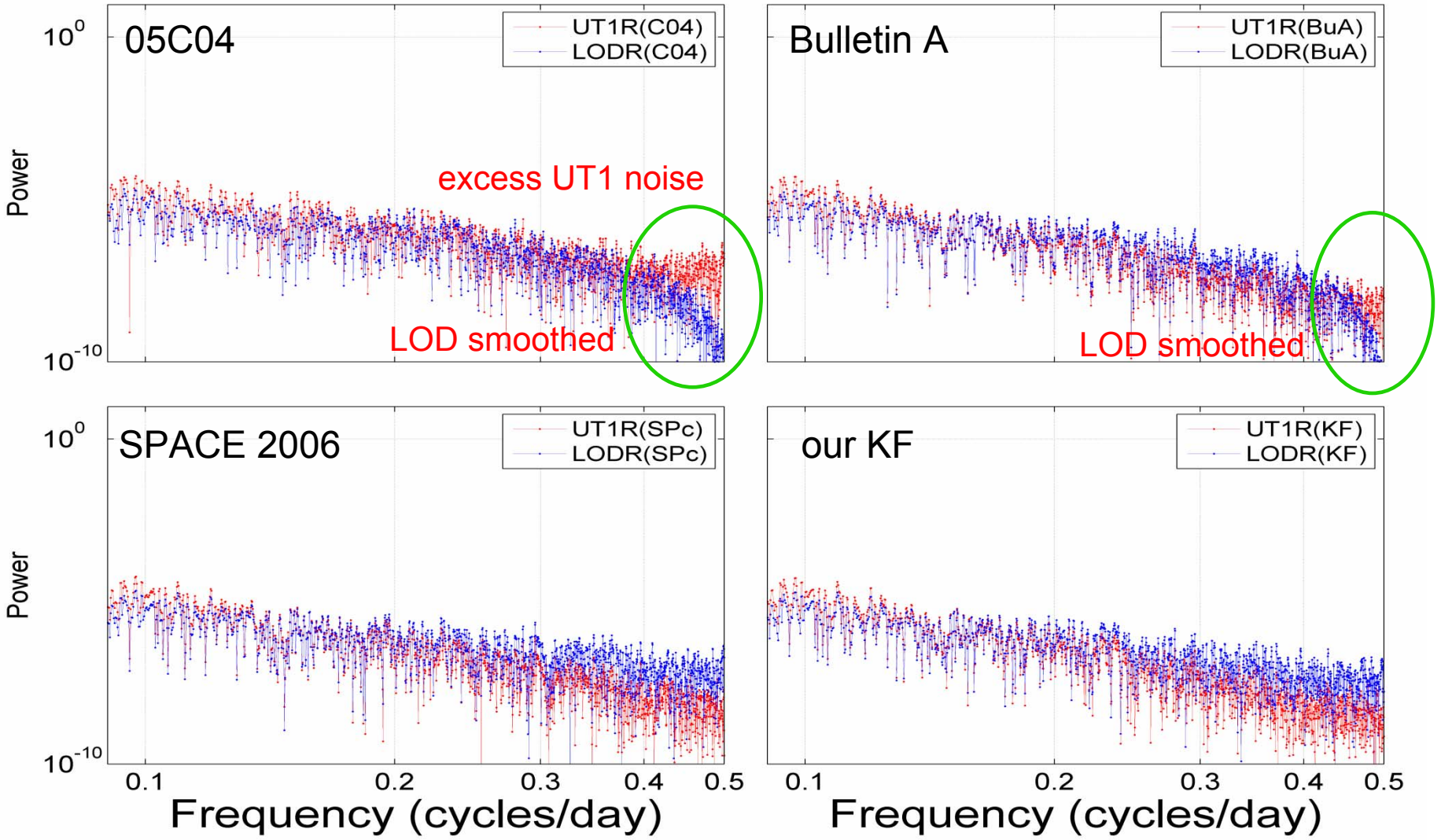
- comparisons w/ AAM excitation only test LOD correlation
- to verify UT1 consistency, compare –dUT1 & LOD values:
  - **05C04**: RMS = **21.2 µs** (**HIGH**)
  - **Bull A**: RMS = **1.9 µs** (**LOW**)
  - **SPC**: RMS = **6.7 µs**
  - **KF**: RMS = **10.5 µs**

expected difference due to –dUT1 interpolation = ~10 µs



UT1 & LOD Power Spectra

- seasonally detrended



Conclusions 1/2

- our KF UT1/LOD combination performs best by all measures
  - further improvements possible, e.g., if VLBI UT1 analyses use IGS polar motion
- IGS LOD series adds critical high-frequency information
  - but care needed to handle correlated biases & spurious signals
- IERS 05C04 LODs correlate well with AAM – based on IGS LODs
  - LODs enjoy benefits & liabilities of IGS LODs; but should filter out spurious signals
  - however, UT1 & LOD values are not consistent
  - strong high-frequency smoothing for LODs; excess noise for UT1
- IERS Bulletin A LODs derived from UT1 values with strong tidal signals
  - LODs have excess high-frequency noise indicated by lower AAM correlations
  - sharp high-frequency smoothing for LODs due to derivation from UT1 values
- SPACE 2006 correlates worst with AAM over all intervals due to excess noise
  - badly hurt by not using GPS LODs

Conclusions 2/2

- our Kalman filter series of UT1 & LOD values is available at:
  - https://goby.nrl.navy.mil/ut1lod/
- MATLAB Kalman filter code is available upon request
  - *Ken.Senior@nrl.navy.mil*
- should IGS produce this KF series as a new product for scientific users ?
  - we prefer that the IERS upgrade its products

Backup Slides

Kalman Filter Combination Model

continuous model

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{w}$$

$$\mathbf{x} = \begin{pmatrix} UT1R \\ LODR \\ M \\ A_f \\ B_f \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} 0 \\ w_L \\ w_M \\ w_{A_f} \\ w_{B_f} \end{pmatrix}$$

$$\mathbf{Q} = E\left[\mathbf{w}\mathbf{w}^T\right] = \begin{pmatrix} 0 & & & & \\ & \sigma_L^2 & & & \\ & & 2\sigma_M^2\beta_M & & \\ & & & \sigma_A^2 & \\ & & & & \sigma_B^2 \end{pmatrix}$$

$$\sigma_L^2 = 0.0036 \text{ msec}^2/\text{day}^3 \qquad \sigma_A^2, \sigma_B^2 \approx 0$$

$$\sigma_M^2 = 0.0282 \text{ msec}^2/\text{day}$$

$$\beta_M = 0.4606 \text{ day}^{-1}$$

VLBI measurements of UT1

$$z_v(t) = H_v\mathbf{x}(t) + \nu_v$$

$$H_v = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

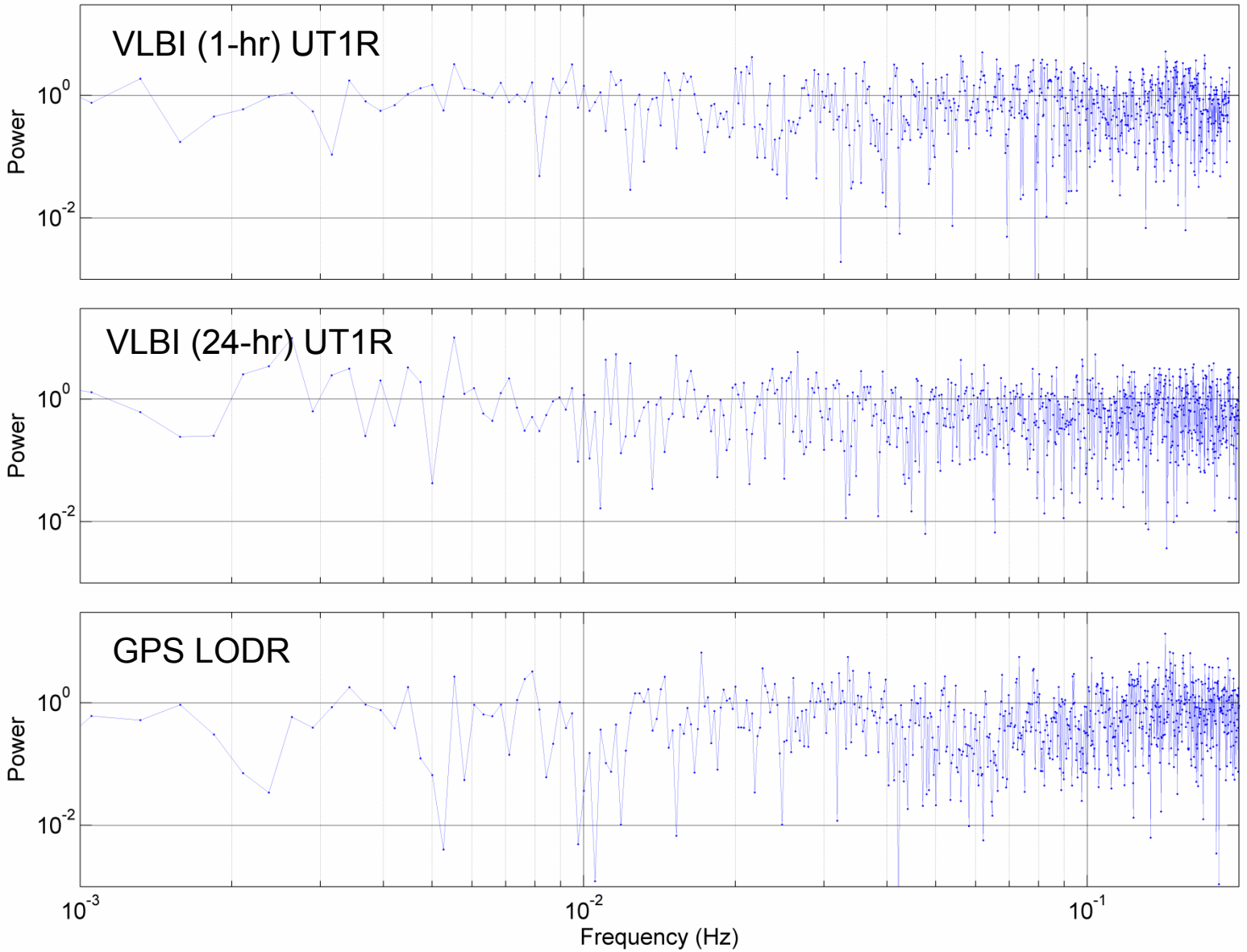
GPS measurements of LOD

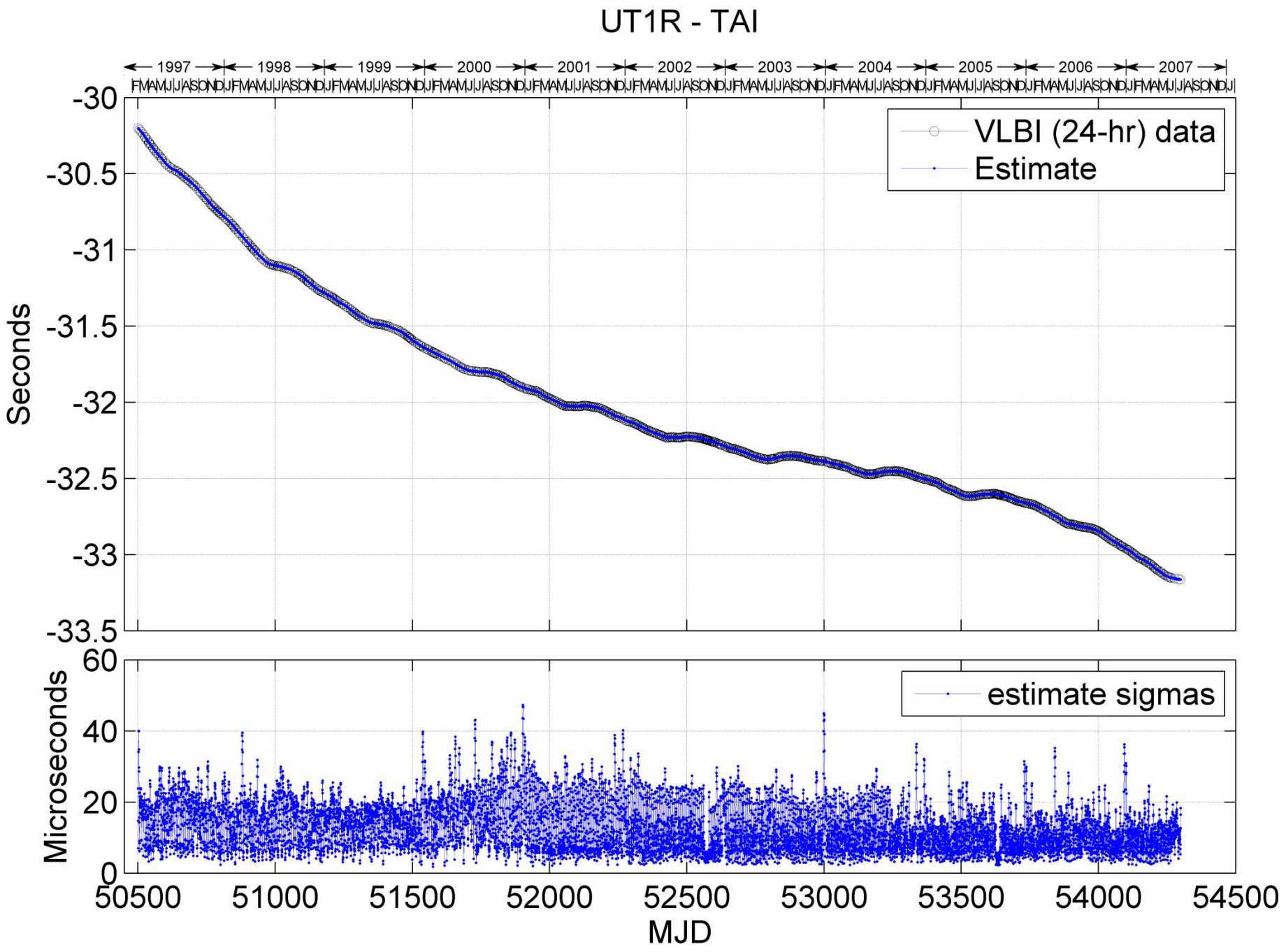
$$z_g(t) = H_g(t)\,\mathbf{x}(t) + \nu_g$$

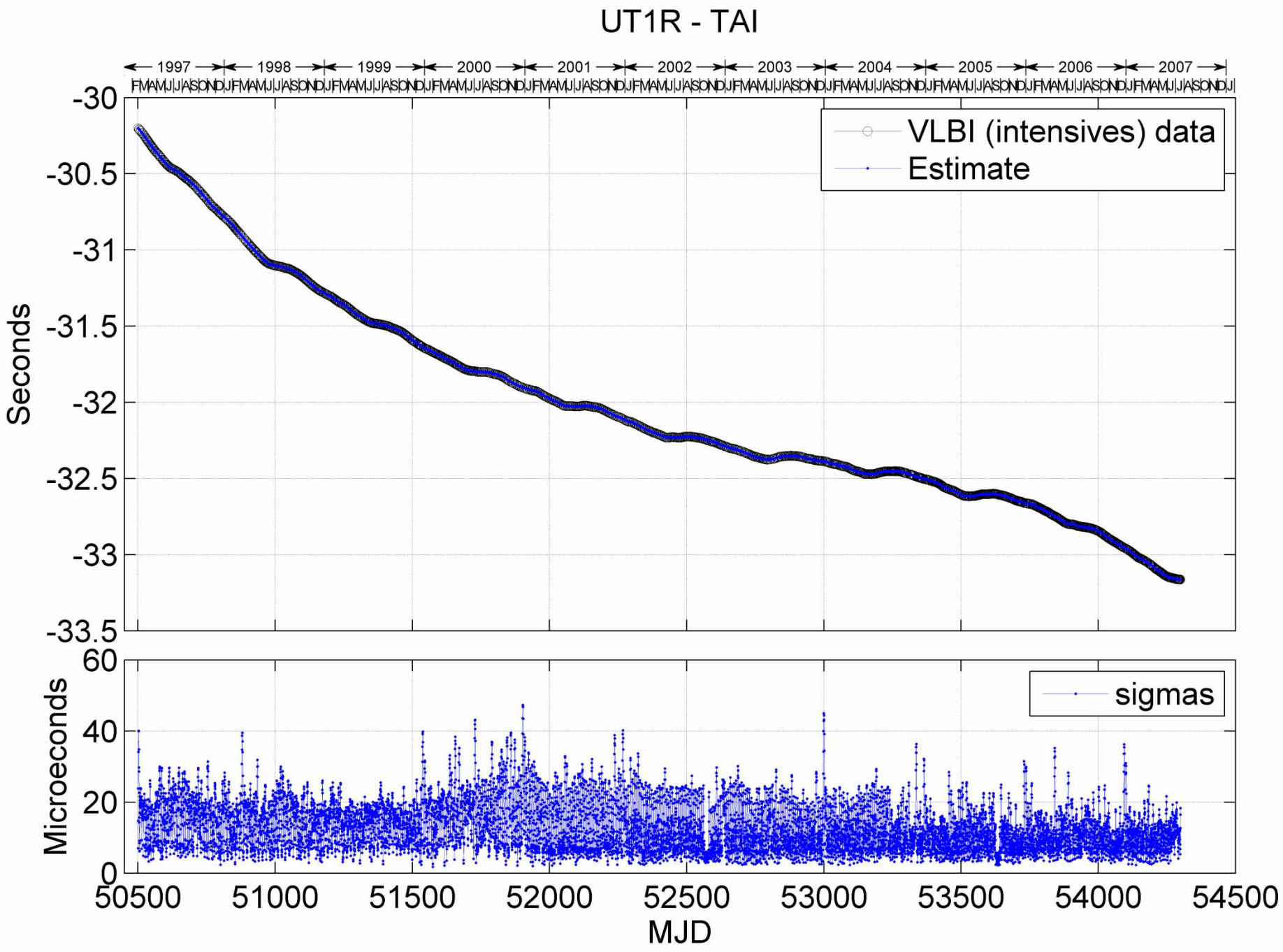
$$H_g(t) = \begin{pmatrix} 0 & 1 & 1 & \cos(2\pi\omega_f t) & -\sin(2\pi\omega_f t) \end{pmatrix}$$

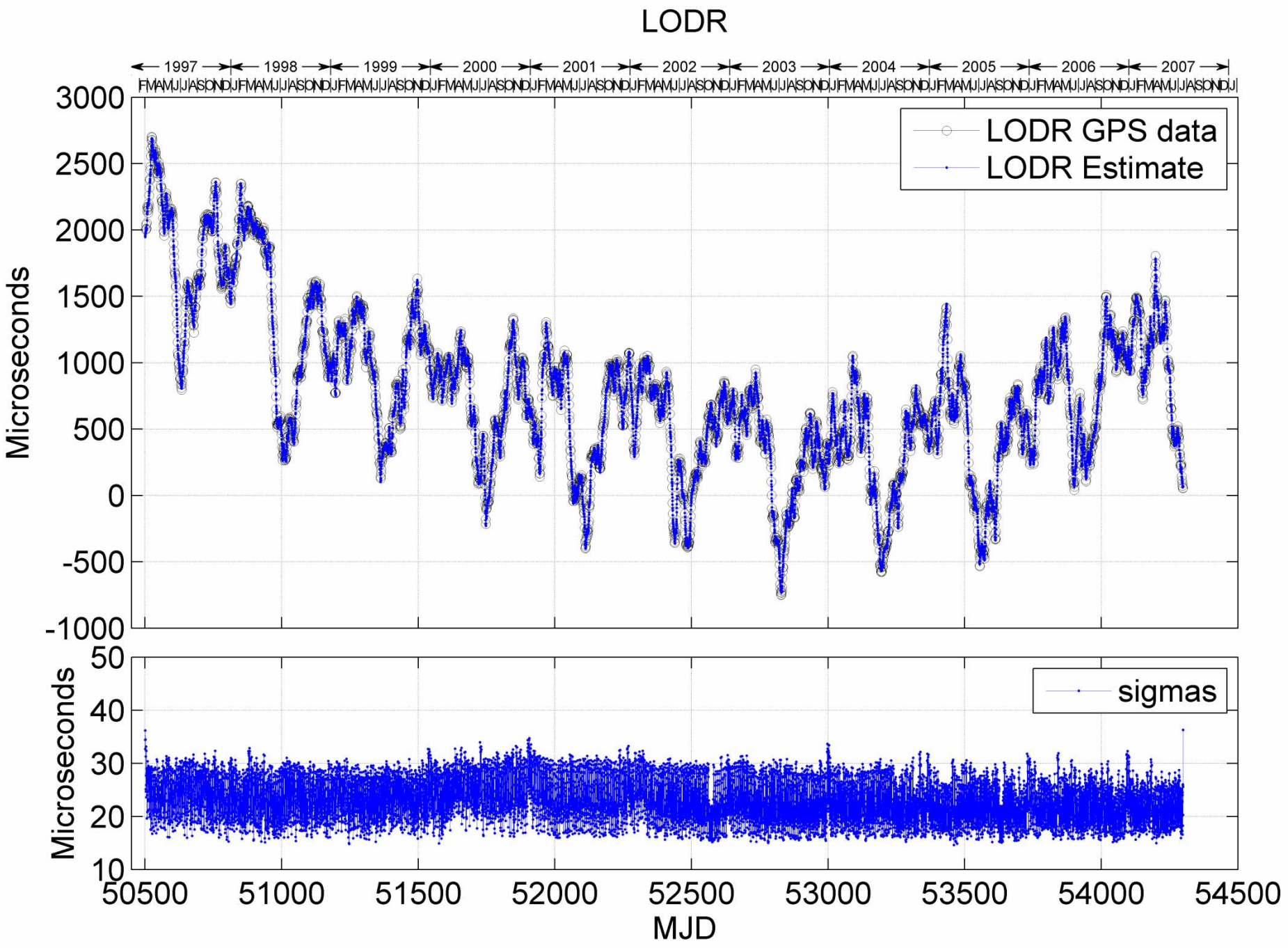
$$\omega_f = 1/14.19 \text{ cycles/day}$$

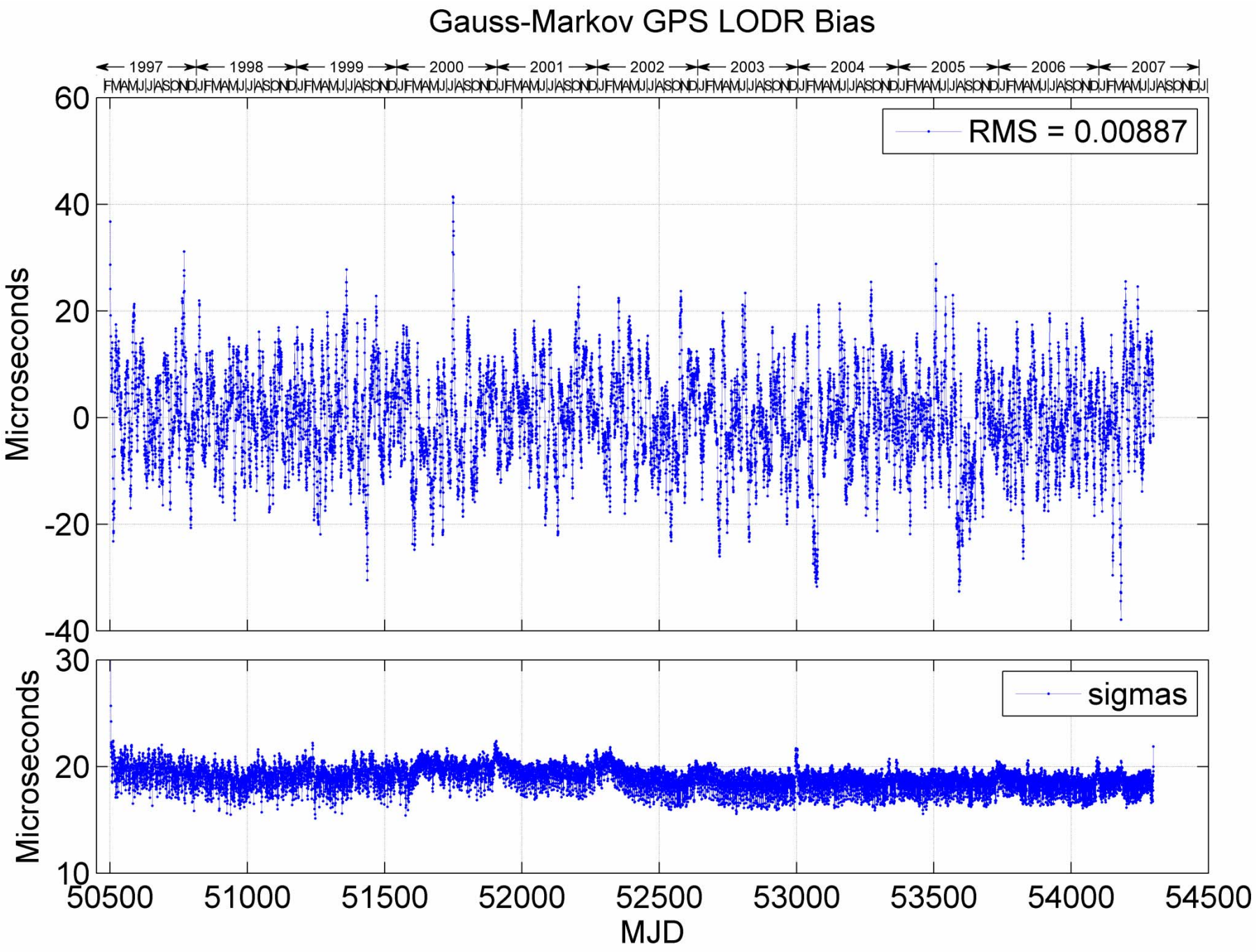
Spectra of Residuals

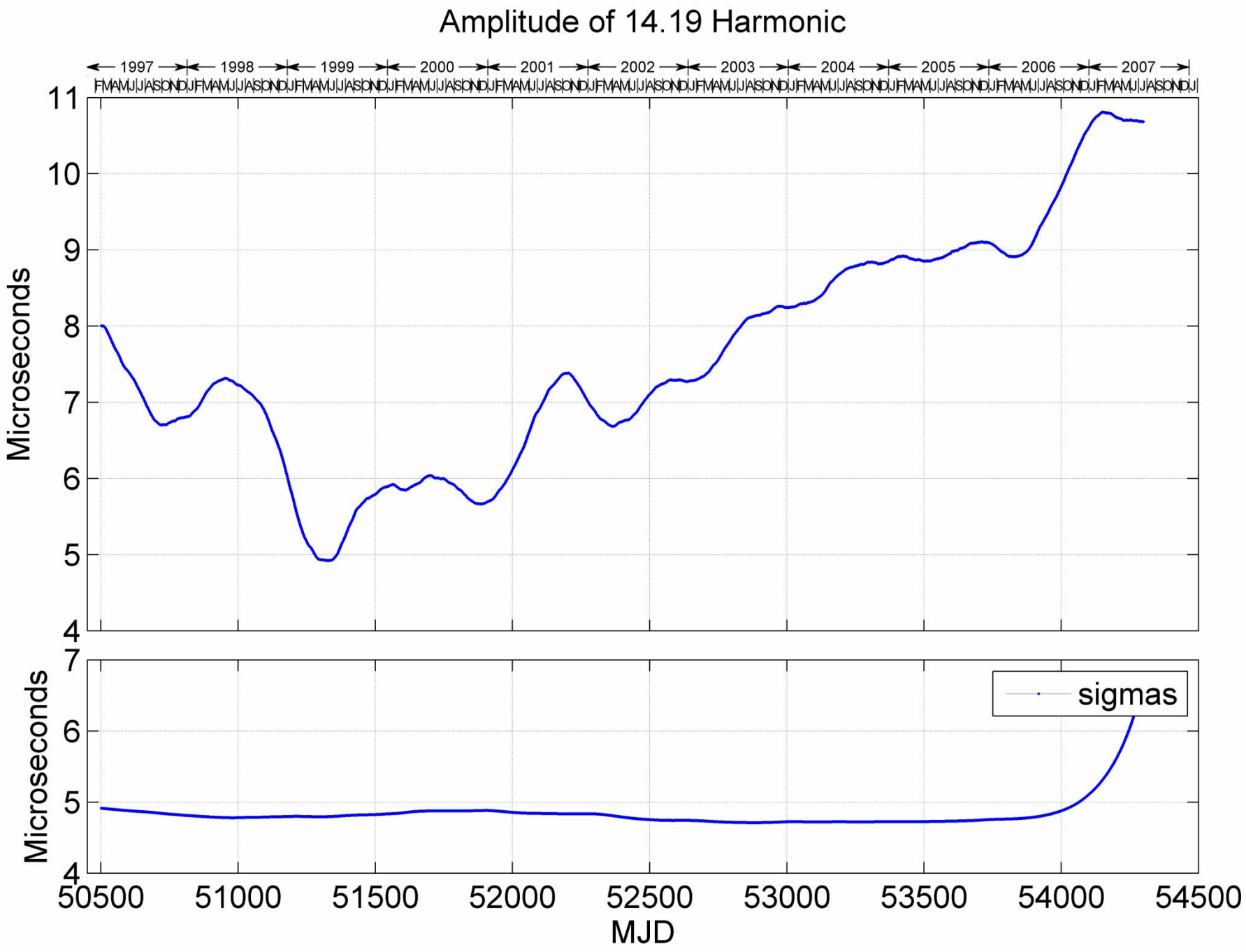


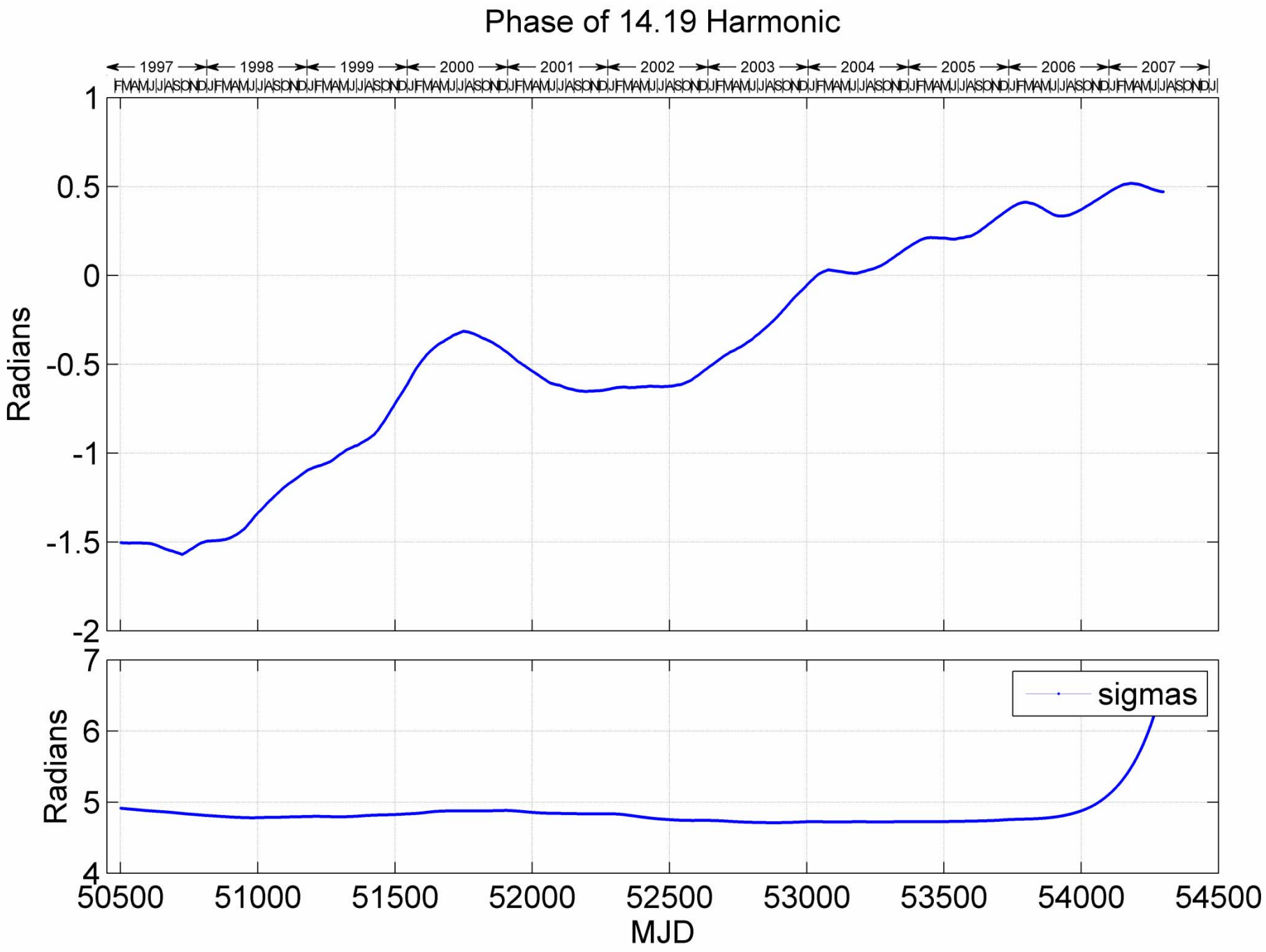

UT1R - TAI
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
VLBI (24-hr) data
Estimate
Seconds
-30
-30.5
-31
-31.5
-32
-32.5
-33
-33.5
estimate sigmas
Microseconds
60
40
20
0
50500
51000
51500
52000
52500
53000
53500
54000
54500
MJD

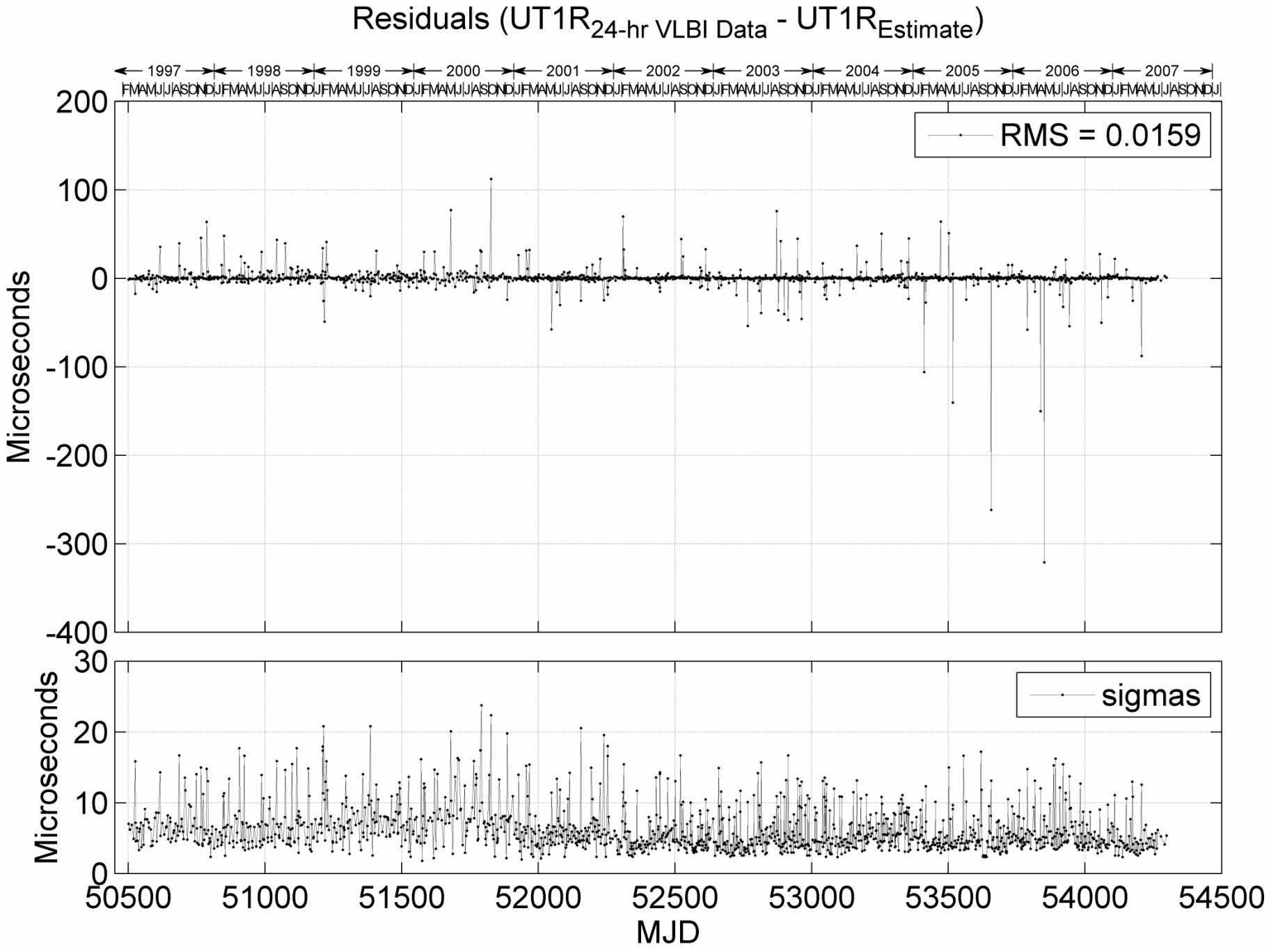

UT1R - TAI
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
VLBI (intensives) data
Estimate
Seconds
-30
-30.5
-31
-31.5
-32
-32.5
-33
-33.5
sigmas
Microseconds
60
40
20
0
50500
51000
51500
52000
52500
53000
53500
54000
54500
MJD

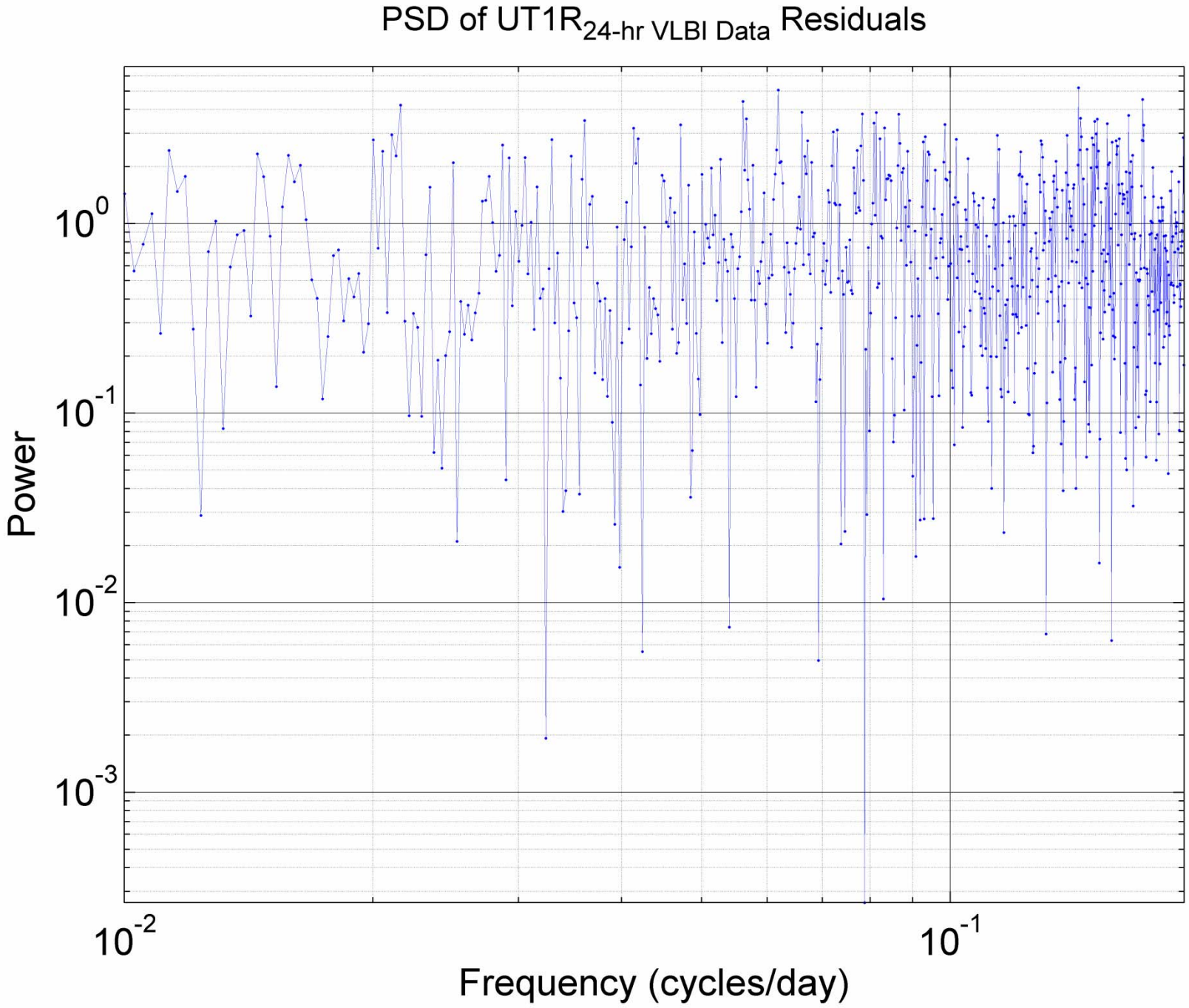

LODR
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
LODR GPS data
LODR Estimate
Microseconds
3000
2500
2000
1500
1000
500
0
-500
-1000
sigmas
Microseconds
50
40
30
20
10
50500
51000
51500
52000
52500
53000
53500
54000
54500
MJD

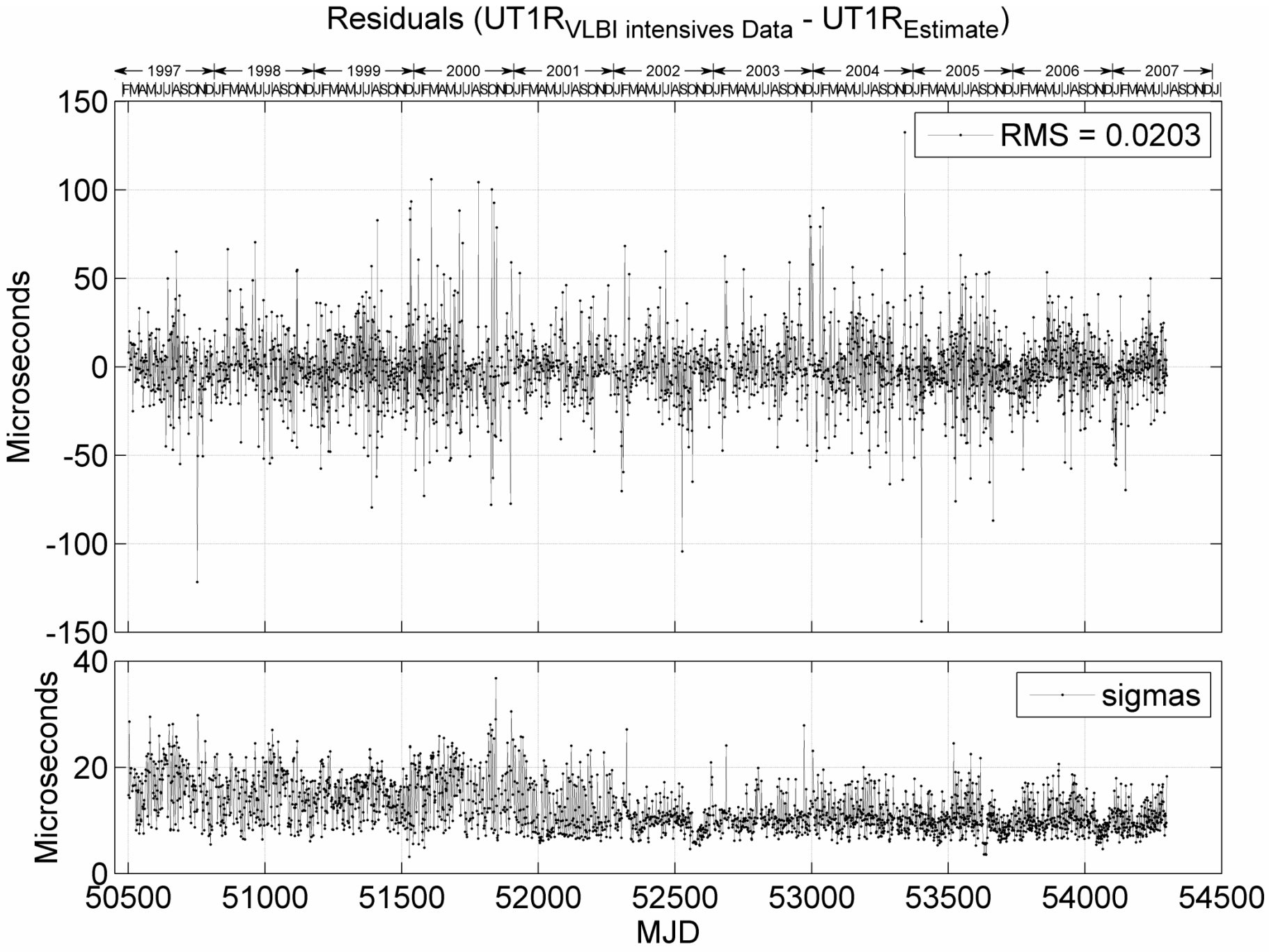

Gauss-Markov GPS LODR Bias
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
RMS = 0.00887
Microseconds
60
40
20
0
-20
-40
sigmas
Microseconds
30
20
10
50500
51000
51500
52000
52500
53000
53500
54000
54500
MJD

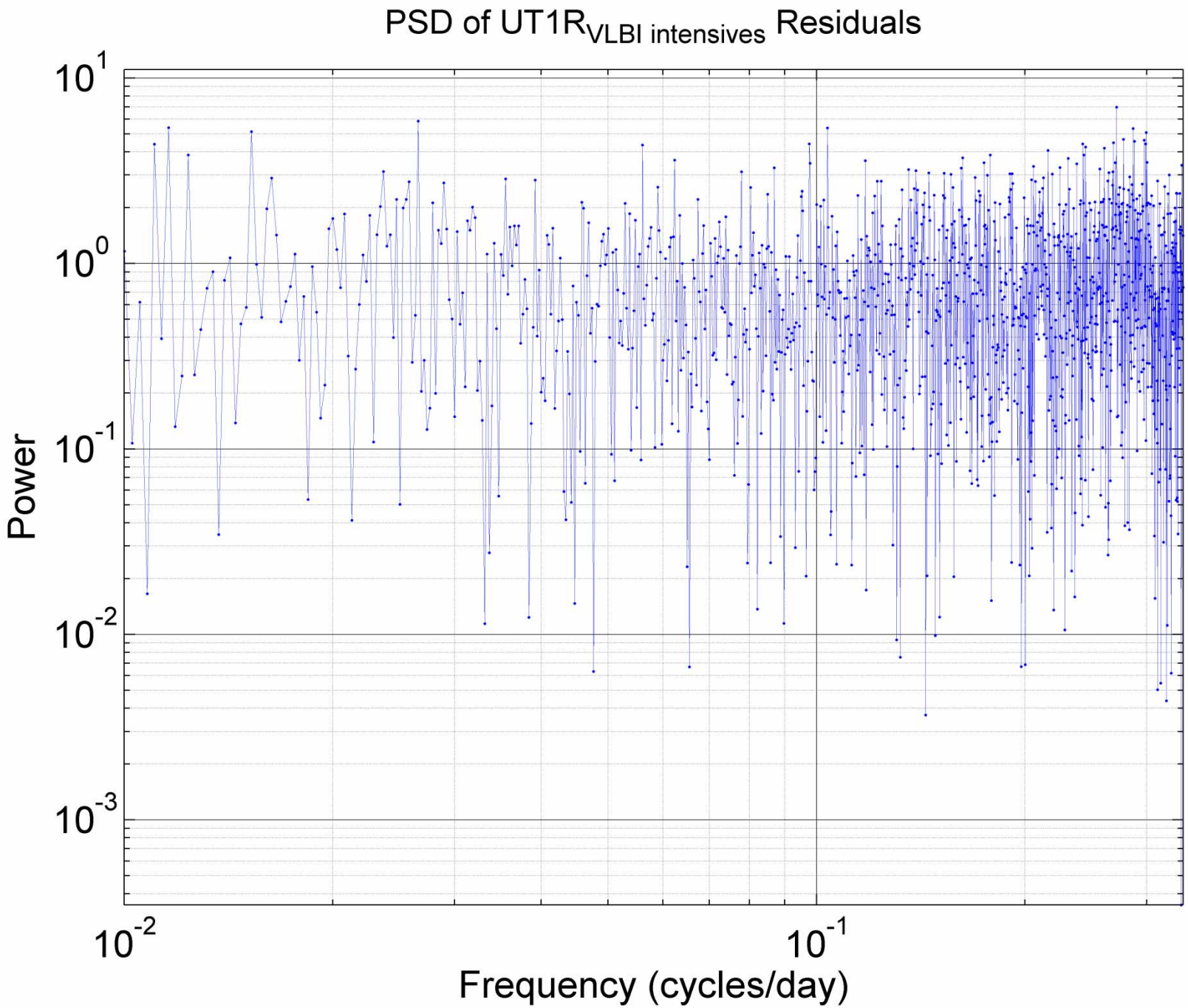

Amplitude of 14.19 Harmonic
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
Microseconds
11
10
9
8
7
6
5
4
Microseconds
7
6
5
4
sigmas
50500
51000
51500
52000
52500
53000
53500
54000
54500
MJD

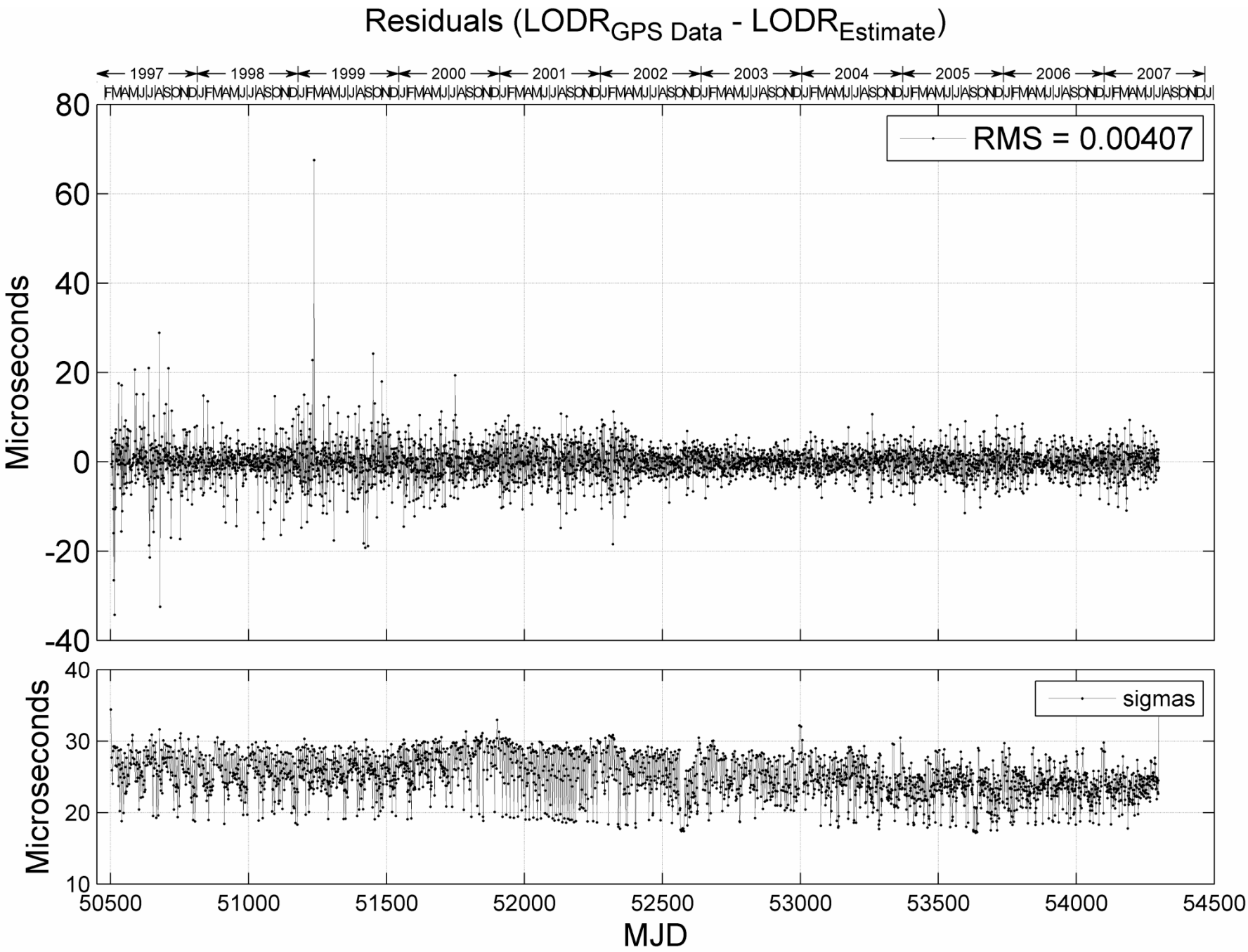

Phase of 14.19 Harmonic
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
Radians
1
0.5
0
-0.5
-1
-1.5
-2
Radians
7
6
5
4
sigmas
50500
51000
51500
52000
52500
53000
53500
54000
54500
MJD

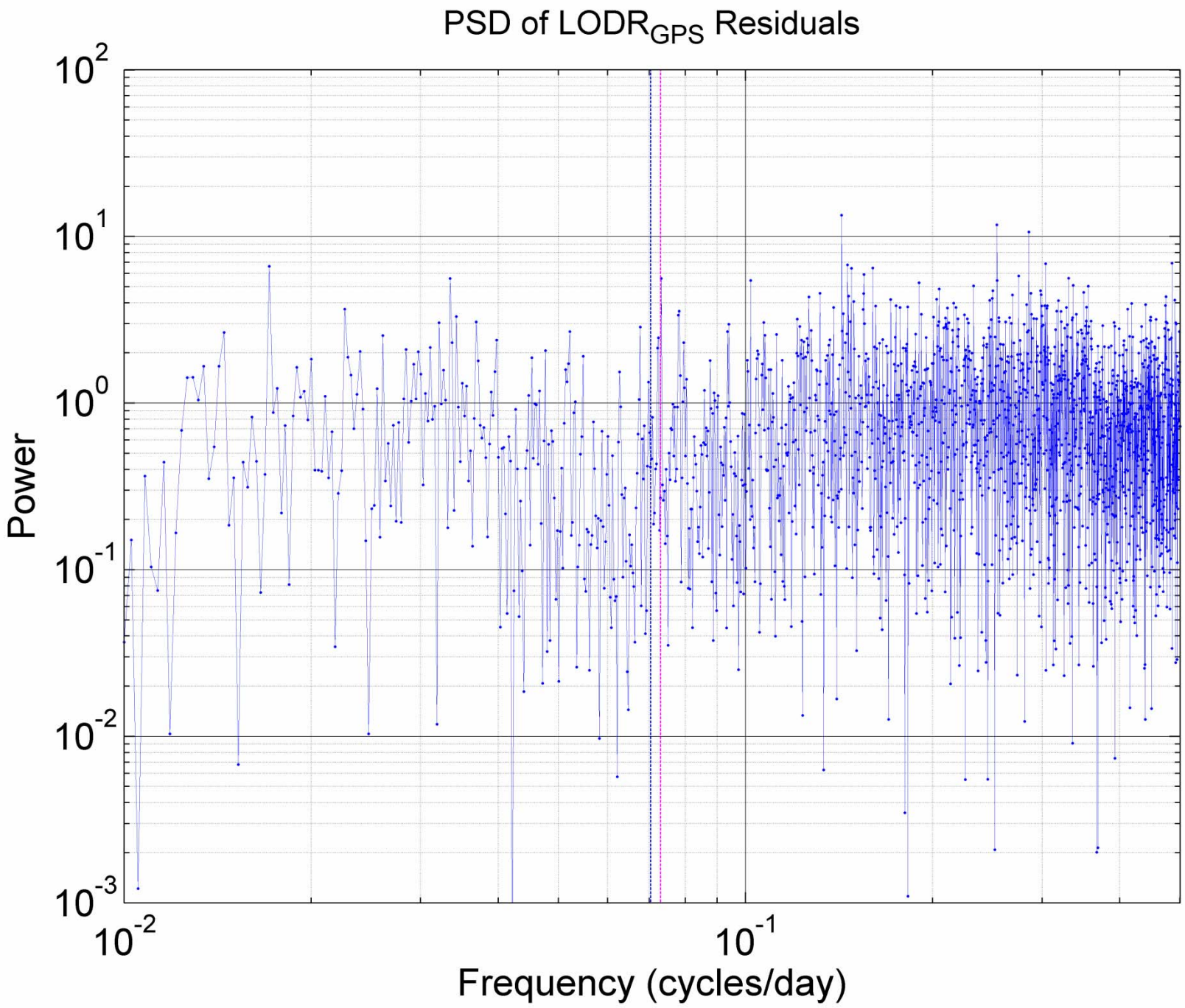

Residuals (UT1R24-hr VLBI Data - UT1REstimate)
1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007
RMS = 0.0159
Microseconds
200
100
0
-100
-200
-300
-400
sigmas
Microseconds
30
20
10
0
50500 51000 51500 52000 52500 53000 53500 54000 54500
MJD


PSD of UT1R24-hr VLBI Data Residuals
Power
10^0
10^-1
10^-2
10^-3
10^-2
10^-1
Frequency (cycles/day)


Residuals (UT1R VLBI intensives Data - UT1R Estimate)
1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007
RMS = 0.0203
Microseconds
150
100
50
0
-50
-100
-150
sigmas
Microseconds
40
20
0
50500 51000 51500 52000 52500 53000 53500 54000 54500
MJD


PSD of UT1R VLBI intensives Residuals
Power
Frequency (cycles/day)
10^1
10^0
10^-1
10^-2
10^-3
10^-2
10^-1


Residuals (LODR GPS Data - LODR Estimate)
1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007
RMS = 0.00407
Microseconds
80
60
40
20
0
-20
-40
sigmas
Microseconds
40
30
20
10
50500 51000 51500 52000 52500 53000 53500 54000 54500
MJD


PSD of LODR$_{GPS}$ Residuals
Power
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
Frequency (cycles/day)

Compare LOD w/ AAM+OAM Excitation

- our KF w/ other LOD combinations
  - corrected for zonal tides, LODS *(Yoder et al., 1981; Kantha et al., 1998)*
- Atmospheric Angular Momentum (AAM) from NCEP Reanalysis
  - 4 values daily, during Feb 1997 – Mar 2006
  - inverted barometer correction applied
  - averaged to daily values at 00:00 or 12:00 epochs to match respective LOD series epochs
- Oceanic Angular Momentum (OAM) from ECCO model *(Gross et al., '05)*
  - 4 values daily, during Feb 1997 – Mar 2006
  - averaged to daily values at 00:00 or 12:00 epochs to match respective LOD series epochs
- for each [LODS – (AAM+OAM)] time series, fit for imperfectly known geophysical & systematic effects *(Kouba & Vondrak, 2005)*
  - annual + semi-annual differences
  - monthly (27.56 d) oceanic tide correction
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- LOD time series studied
  - IERS 05C04 (00:00)
  - IERS Bulletin A (00:00)
  - JPL's SPACE 2006 (12:00)
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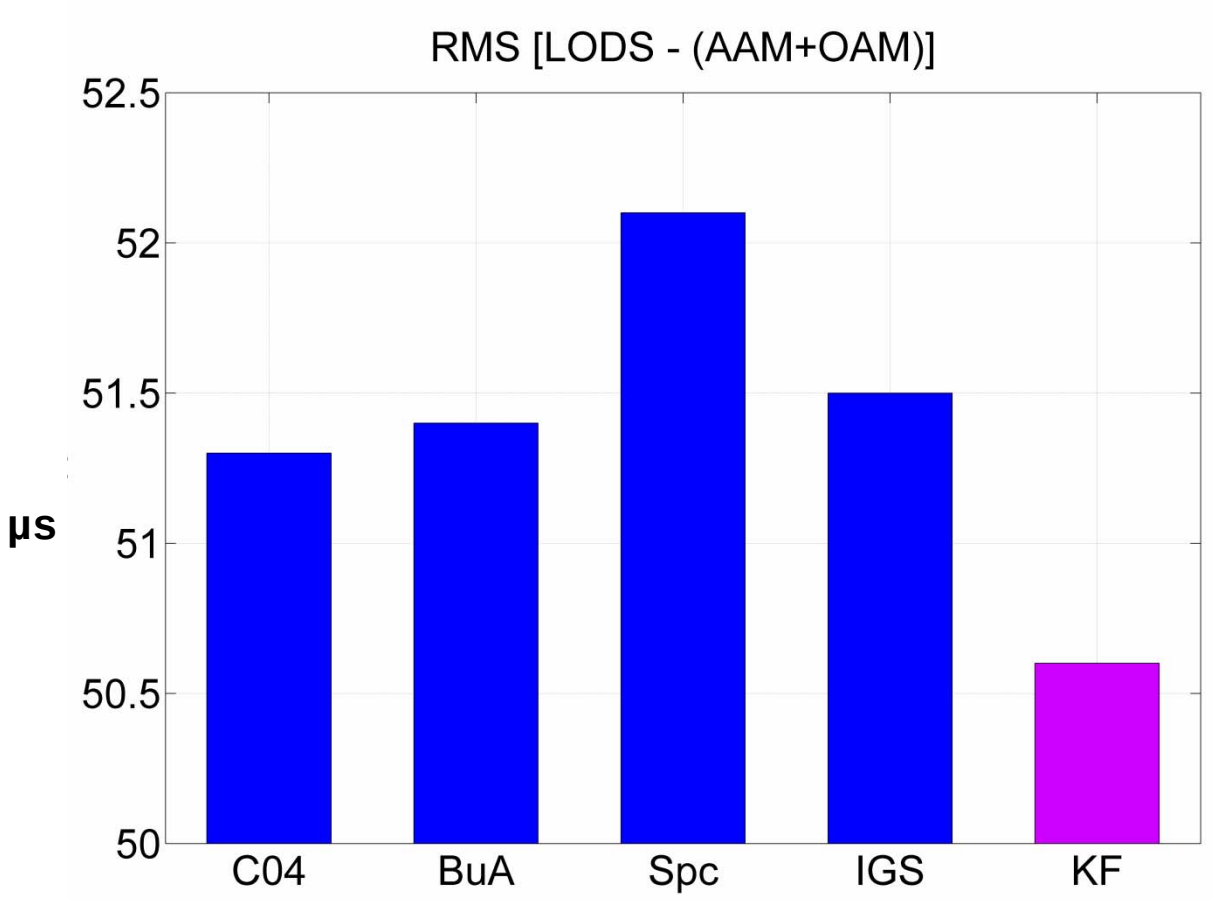
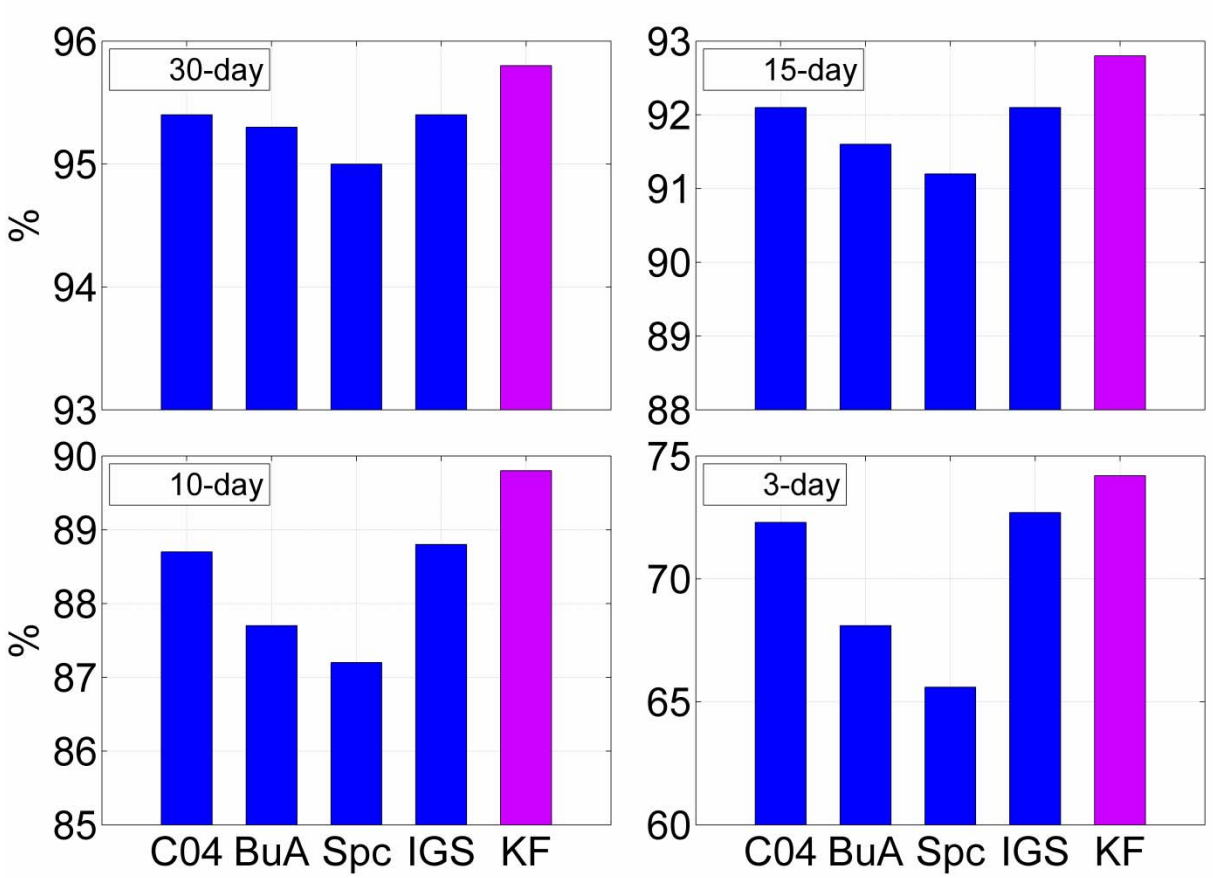
  - our KF (12:00)

- our KF has smallest residual



LODS / (AAM+OAM) Correlation Coefficients

- computed over sliding windows from 3 d to 5.6 yr
  - correlation over full range = 99.0% for BuA, C04, SPc, & IGS and 99.1% for KF



- KF has highest correlations w/ AAM+OAM over all intervals; SPACE 2006 has lowest

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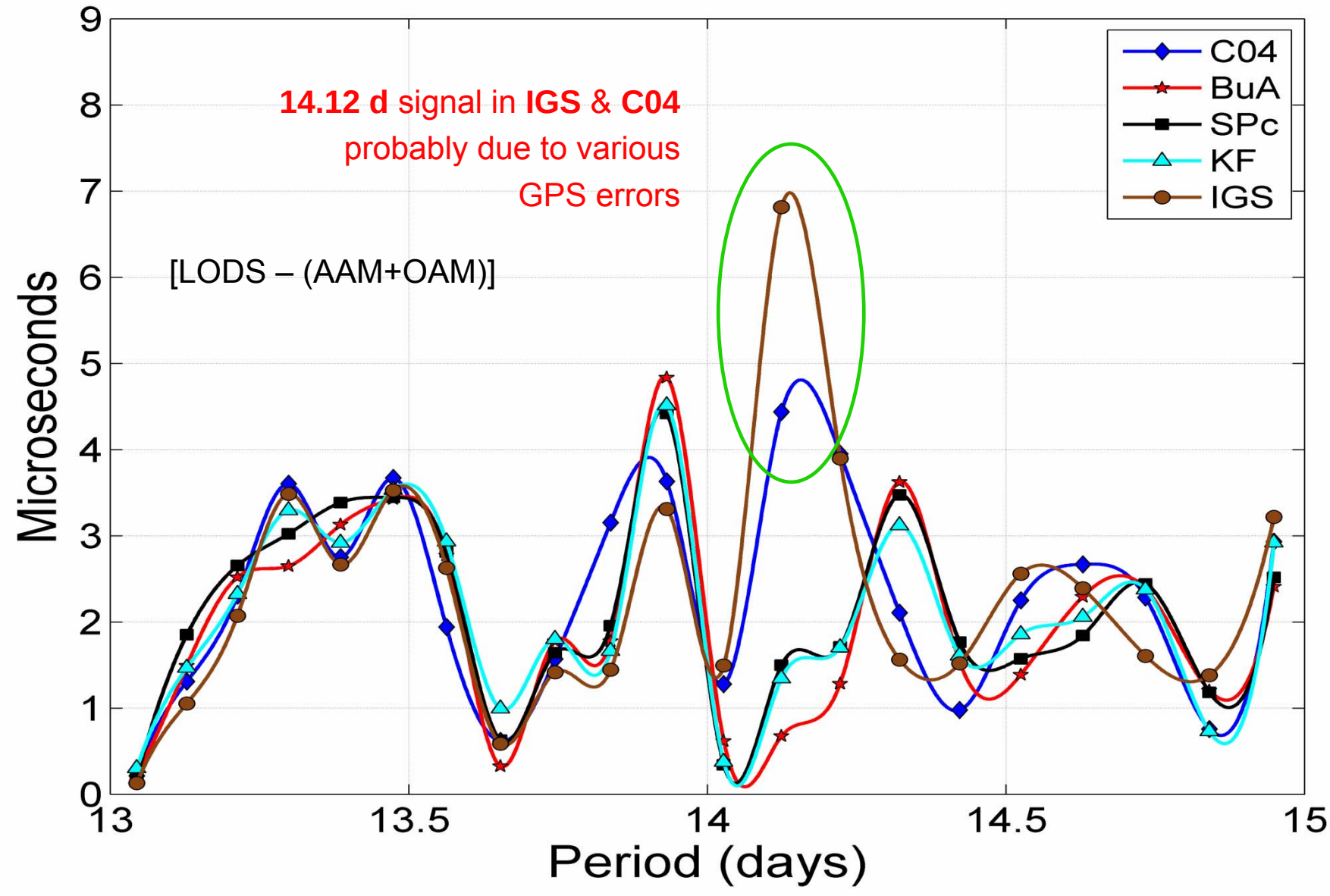
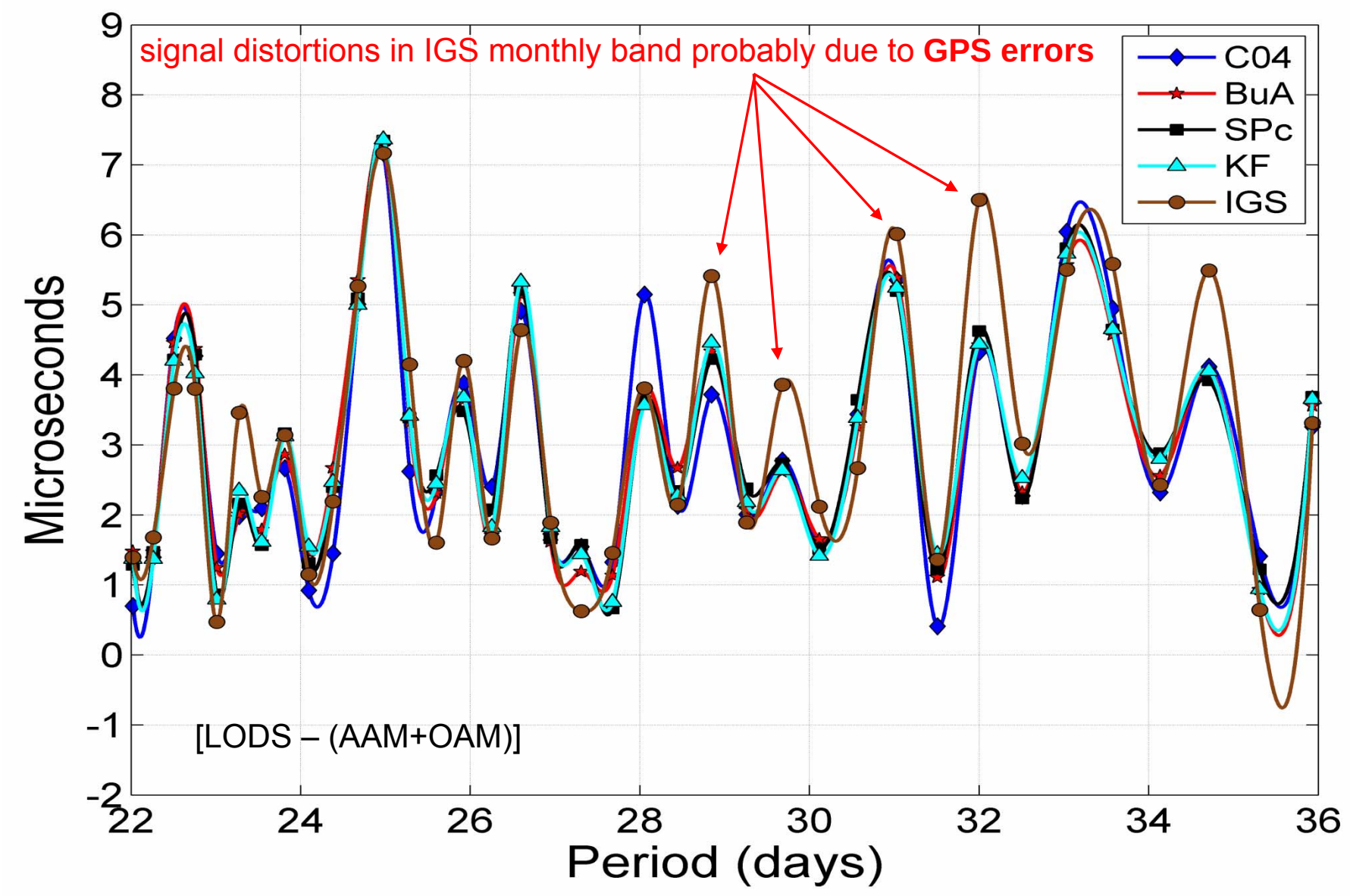
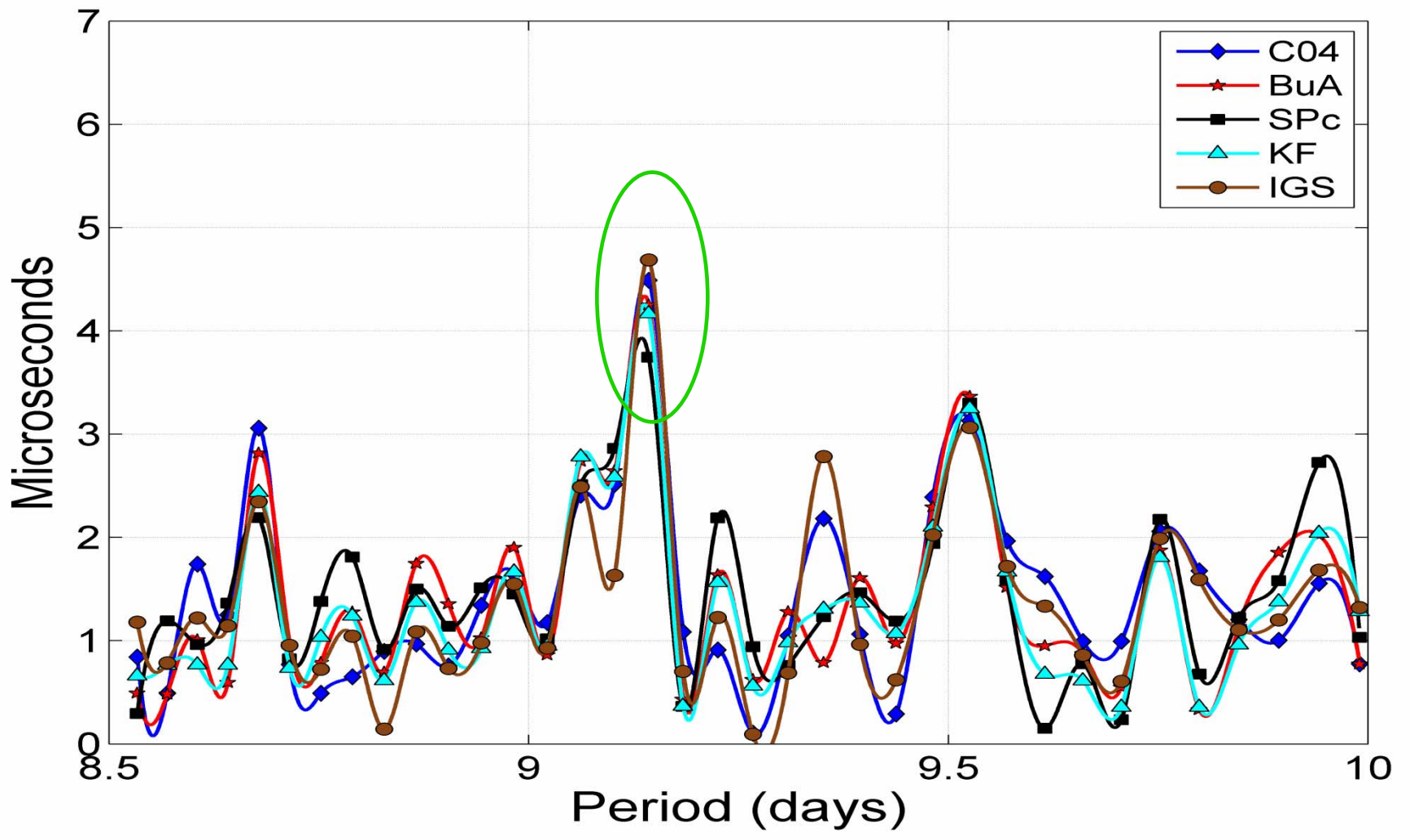


Monthly Band – Probable GPS Errors



9-d Band – Unmodeled Geophysical Effect?



- 9.14 d peak seen in all LODS series – probably geophysical
- needs further investigation – could be included in [LODS-(AAM+OAM)] fit